[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 8749 **C** **Roll No............**

Unique Paper Code : 235372

Name of the Paper : MAHT-305 : Mathematics–II

Name of the Course : B.Sc. (Hons.) Electronics, Part II (Admissions of 2010 and onwards)

Semester : III

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Use of Scientific Calculator is allowed.
3. There are two sections. **Section I** is compulsory.
4. Attempt any **four** questions from **Section II**. Marks are as indicated.

SECTION I

1. (a) Let T be a transformation from $\mathbb{R}^3$ into $\mathbb{R}^1$ define by

$$T(x_{1,}x_{2,}x_{3,}) = x_1^2 + x_2^2 + x_3^2$$

Show that T is not a linear transformation.

(b) Find the rank of A, where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

(c) Prove that the eigen values of Unitary matrix U have absolute value 1.

(d) Determine and classify all singularities of the given functions :

$$\frac{z^8 + z^4 + 2}{(z-1)^3(z^2+4)^4(3z-5)^5}$$

(e) Obtain the differential equation of the family of parabolas with foci at the origin and axes along the x-axis. (3×5)

SECTION II

2. (a) Find the values of a and b for which the system :

$$x + y + z = 6,$$
$$x + 2y + 3z = 10,$$
$$x + 2y + az = b.$$

has (i) no solution (ii) unique solution (iii) infinite number of solutions.

(b) Find the values of k for which the set of vectors

$$\{(k,1-k,k),(0,3k-1,2),(-k,1,0)\}$$

form a basis in $\mathbb{R}^3$.

(c) Solve the systems of equations :

$$x + 2y - 2z = 1$$
$$2x - 3y + z = 0$$
$$5x + y - 5z = 1$$
$$3x + 14y - 12z = 5,$$

using Gauss elimination method. (5×3)

3. (a) Find the eigen values and eigen vectors of :

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}.$$

(b) Use Cayley-Hamilton theorem to find A^{-1} if

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

(c) Diagonalize

$$A=\begin{bmatrix}1 & 1 & 3\\ 1 & 5 & 1\\ 3 & 1 & 1\end{bmatrix}$$

and hence find A^4. Find the modal matrix. (5×3)

4. (a) Determine the analytic function f(z) such that :

$\text{Re}\left(f'(z)\right)=3x^2-4y-3y^2$ and $f(1+i)=0$.

(b) Find all the solutions of $\sin z = 3$.

(c) Find the Taylor's series expansion of f(z). Determine the region of convergence

$$f(z)=\frac{a}{bz+c} \text{ about } z=1.$$

(5×3)

5. (a) Solve the following : $\left(2xy+3y^2\right)dx-\left(2xy+x^2\right)dy=0$

(b) Determine for what value of a and b, the following differential equation is exact and obtain the general solution of the exact equation :

$$\left(ay+bx^3\right)dx+\left(x+y^3\right)dy=0$$

(c) Show that the family of parabolas $y^2 = 2cx + c^2$ is "self-orthogonal".

(5×3)

6. (a) Evaluate :

$$\oint_C \left(e^{-1/z}\sin\left(\frac{1}{z}\right)\right)dz$$

where C is the circle $|z| = 1$.

(b) Solve the system of equations

$$\begin{aligned} 2x_1 - 2x_2 - 2x_3 &= -4\\ -2x_2 + 2x_3 &= -2\\ -x_1 + 5x_2 + 2x_3 &= 6 \end{aligned}$$

by LU-decomposition.

(c) Determine the equation of the tangent, equation of the normal, lengths of the tangent, normal, sub-tangent and subnormal to the curve $y = 3x^2$. (5×3)

7. (a) Solve :

$$\cos x \, dy = y(\sin x - y)dx.$$

(b) Evaluate :

$$\int_0^{2\pi} \frac{d\theta}{\sqrt{2} - \cos\theta}$$

(c) Show that :

$$A = \begin{bmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$$

is Skew-Hermitian and also unitary. Find the eigen values and eigen vectors. (5×3)