[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 6454 **D** **Your Roll No................**

Unique Paper Code : 235372

Name of the Course : **B.Sc. (Hons.) Electronics – II**

Name of the Paper : Mathematics – II / Code : MAHT 305

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Use of Scientific Calculator is allowed.
3. There are **two** sections. **Section I** is compulsory.
4. Attempt any **four** questions from **Section II.** Marks are as indicated.

SECTION I

1. (a) Show that $S = \{(1,0,0), (1,1,0), (1,1,1), (0,1,0)\}$ is a spanning set of $\mathbb{R}^3$ but is not a basis of $\mathbb{R}^3$.

(b) Find the rank of the matrix whose column vectors are

$$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$$

(c) If A and B are square matrices of the same order and A is symmetric, then prove that B^TAB is also symmetric.

(d) Determine the singularities and name the type of singularity :

$$\frac{z}{e^z - 1}$$

P.T.O.

(e) Obtain differential equation of family of plane curves which are all circles of unit radius. (3×5=15)

SECTION II

2. (a) Given $H = \{(5b + 2c, b, c): b, c \in \mathbb{R}\}$, show that H is a subspace of $\mathbb{R}^3$. Find a basis and dimension of H.

(b) Find the values of a and b for which the system

$$2x + 3y + 5z = 9$$

$$7x + 3y - 2z = 8$$

$$2x + 3y + az = b$$

has (i) no solution (ii) unique solution (iii) infinitely many solutions. Find the solutions in case (iii).

(c) Solve the following system of equations by Gaussian elimination method.

$$2x_1 + x_2 + 4x_3 = 12$$

$$8x_1 - 3x_2 + 2x_3 = 20$$

$$4x_1 + 11x_2 - x_3 = 33$$

(5×3=15)

3. (a) Given $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$, find A^{-1} using Cayley Hamilton Theorem.

Also find $B = A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$.

(b) Diagnolize the given matrix $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$ and hence find A^2. Find the modal matrix P which diagnolizes A.

(c) Find the nature, index and signature of the quadratic form :

$$2x_1x_2 + 2x_1x_3 + 2x_2x_3$$

(5×3=15)

4. (a) If f is analytic show that $f' = (\cos\theta - i\sin\theta)\frac{\partial f}{\partial r}$.

(b) Integrate the given function around the given contour C :

$$\frac{\sin z}{z^2 - iz + 2}$$

where C: rectangle with vertices at (1, 0), (1, 3), (–1, 3) and (–1, 0).

(c) Evaluate $\int_0^{2\pi} \frac{1}{5 - 3\sin\theta} d\theta$. (5×3=15)

5. (a) Solve $(2x - 5y)dx + (4x - y)dy = 0$; $y(1) = 4$.

(b) The slope at any point (x, y) of a curve is $1 + \frac{y}{x}$. If the curve passes through (1, 1), find it's equation.

(c) Solve $(y - x^3)dx + (x + y^3)dy = 0$. (5×3=15)

6. (a) Solve AX = B by LU-decomposition using Gaussian elimination where

$$A = \begin{pmatrix} 2 & 4 & -6 \\ 1 & 5 & 3 \\ 1 & 3 & 2 \end{pmatrix}$$

and $B^T = (-4, 10, 5)$.

(b) Prove that $A = \begin{bmatrix} 4 & 1 - 3i \\ 1 + 3i & 7 \end{bmatrix}$ is Hermitian. Find its eigen values.

(c) Show that for :

$$f(z) = \begin{cases} \frac{2xy(x + iy)}{x^2 + y^2} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$$

the CR equations are satisfied at origin but derivatives of f(z) at origin does not exist. (5×3=15)

7. (a) Solve $\frac{dy}{dx} = \frac{1}{1 + x^2}\left(e^{\tan^{-1} x} - y\right)$.

(b) Find all solutions of $i \sinh iz = 2$.

(c) Find particular member of orthogonal trajectories of $x^2 + cy^2 = 1$ passing through the point (2,1). (5×3=15)