This question paper contains 3 printed pages. *Your Roll No.* ....

Sl. No. of Ques. Paper	: 952	G
Unique Paper Code	: 235372	
Name of Paper	: MAHT–305 : Mathematics – II	
Name of Course	: B.Sc. (Hons.) Electronics	
Semester	: III	
Duration	: 3 hours	
Maximum Marks	: 75	

(Write your Roll No. on the top immediately on receipt of this question paper.)

Use of scientific calculator is allowed. There are two Sections. Section I is compulsory. Attempt any four questions from Section II.

SECTION I

1. (a) Determine a polynomial p such that $p=aq+br$, where:

$$q(x)=x-2x^3+3x^4,\ r(x)=3-x+3x^2+2x^3-3x^4,\ a=3,\ b=3$$ 3

(b) Find the rank of the matrix:—

$$\begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ 2 & 3 & 10 \end{bmatrix}$$ 3

(c) Show that $f(z)=\overline{z}$ is continuous but not differentiable. 3

(d) Find the differential equation of the family of parabolas with foci at the origin and axes along x axis. 3

(e) Find all the roots of the equation $\tanh z+2=0$. 3

SECTION II

2. (a) Show that the vectors $b_1=(1,1,1)$, $b_2=(1,0,-1)$ and $b_3=(1,-2,1)$ are mutually orthogonal. Find the length of each given vector. Express $(1,0,5)$ as a linear combination of b_1, b_2 and b_3. 5

(b) Solve the system by LU-decomposition:

$$3x-6y-3z=-3,$$
$$2x+6z=-22,$$
$$-4x+7y+4z=3$$ 5

(c) Define Linear Transformation. Show that the function T defined on R^2 by :

$$T(x,y)=3x-5y$$

is a linear transformation from R^2 into R. 5

3. (a) Verify Cayley-Hamilton theorem for the matrix:

$$A=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 5\\ 3 & 5 & 6\end{bmatrix}$$

and hence find the inverse of A. 5

(b) Show that:

$$A=\begin{bmatrix}i & 0 & 0\\ 0 & 0 & i\\ 0 & i & 0\end{bmatrix}$$

is Skew-Hermitian and also unitary. Find the eigen values and eigen vectors. 5

(c) Find a matrix P which diagonalizes the matrix:

$$A=\begin{bmatrix}4 & 1\\ 2 & 3\end{bmatrix}$$

Verify that $P^{-1}AP=D$ where D is the diagonal matrix. Hence find A^6. 5

4. (a) Solve:

$$\left(x\cos\frac{y}{x}+y\sin\frac{y}{x}\right)y-\left(y\sin\frac{y}{x}-x\cos\frac{y}{x}\right)x\frac{dy}{dx}=0$$ 5

(b) Solve:

$$(x-y)\,dx-dy=0,\ y(0)=2$$ 5

(c) Show that the family of parabolas $y^2=4cx+c^2$ is "self orthogonal". 5

5. (a) Find the Taylor's series expansion of $f(z)$. Determine the region of convergence

$$f(z)=\frac{1}{(2z+1)^2}$$

about $z=2$. 5

(b) Evaluate :

$$\int_0^{\pi}\sin^4\theta\, d\theta.$$ 5

(c) Determine the analytic function $f(z)$ such that:

$$\operatorname{Im}(f'(z))=6x(2y-1) \text{ and } f(0)=3-2i,\ f(1)=6-5i$$

Find $f(1+i)$. 5

6. (a) Determine the values of a and b for which the system:

$$x+y+z=6,$$
$$x+2y+3z=10,$$
$$x+2y+az=b$$

has (i) no solution, (ii) unique solution and (iii) infinite number of solutions. 5

(b) If $f(z)=u+iv$ is an analytic function of $z=x+iy$ and

$$u+v=(x+y)(2-4xy+x^2+y^2)$$

then find u, v and the analytic function $f(z)$. 5

(c) Solve $y^2\,dx+(3xy-1)\,dy=0$. 5

7. (a) Solve the system by Gauss Jordan method:

$$x+y+z=6,$$
$$2x-3y+4z=8,$$
$$x-y+2z=5.$$

5

(b) Find the Laurent series of $f(z)=\dfrac{e^z}{z(1-z)}$ about $z=1$. Find region of convergence. 5

(c) Solve $y(x^3e^{xy}-y)\,dx+x(xy+x^3e^{xy})\,dy=0$. 5