[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1820 **C** **Roll No............**

Unique Paper Code : 251606

Name of the Course : B.Sc. (H) Electronics

Name of the Paper : ELHT 604 : Engineering Mathematics

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all including Question No. **1** which is compulsory.
3. Use of non-programmable scientific calculator is allowed.

1. Attempt any **five** :

(a) Solve the differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^x$$

(b) Find the eigenvalues and eigenfunctions of the Sturm-Liouville problem

$$\frac{d^2y}{dx^2} + \lambda y = 0 \text{ with } y(0) = 0 \text{ and } y(\pi) = 0$$

(c) List nature of solution for second order linear-homogeneous differential equation with constant coefficients for

(i) two real and distinct roots

(ii) real and equal roots

(iii) complex conjugate roots

(d) Show that sin2x and cos2x are solutions of differential equation $y'' + 4y = 0$ and that these solutions are linearly independent.

(e) Use the method of separation of variables to solve the initial value problem

$$4u_x + u_y = 3u \quad \text{with } u(0, y) = e^{-5y}$$

where $u_x = \dfrac{\partial u}{\partial x}$, $u_y = \dfrac{\partial u}{\partial y}$

(f) Prove the orthogonality of Legendre's polynomial

$$\int_{-1}^{+1} P_m(x)\, P_n(x)\, dx = 0$$ (5×3)

2\. (a) Solve

$$\left(\frac{d^2y}{dx^2}\right) - 4\left(\frac{dy}{dx}\right) - 5y = e^{2x} + 3\cos(4x + 3)$$

(b) Consider a circuit of a resistance (R = 5 ohms), an inductor (L = 0.05H) and a capacitor ($C = 4\times10^{-4}$ Farads) connected in series. If the charge, 'Q' and current, 'I' are zero when t = 0, find Q(t) and I(t) when there is a constant emf of 110 volts. (6+9)

3\. (a) Solve

$$\frac{d^2y}{dx^2} + y = 0$$

using power series method and obtain the recurrence relation. Comment and compare with the solution obtained with the standard solution of this equation.

(b) Using the power series solution obtained in part (a), show that the two solutions obtained are linearly independent at x = 0.

(c) Prove that $J_n(x)$ and $J_{-n}(x)$ are linearly independent, i.e.

$$J_{-n}(x) = (-1)^n J_n(x)$$ (7+4+4)

4. (a) Derive the d'Alembert's solution of the one dimensional wave equation.

(b) Form a partial differential equation by eliminating the arbitrary constant a, b from the relation

$$x^2 + y^2 + (z-a)^2 = b^2$$

where z is the dependent variable and x, y are the independent variables.

(c) Solve the partial differential equation

$$z_x - z_y = \ln(x + y)$$

where $z_x = \frac{\partial z}{\partial x}$, $z_y = \frac{\partial z}{\partial y}$

z is the dependent variable and x, y are the independent variables.

(3+6+6)

5. Considering the motion of a stretched elastic vibrating membrane in the xy-plane, determine the solution $u_{m,n}(x,y,t)$ of the two-dimensional wave equation for small transverse vibration if 'ρ' is the mass of the membrane per unit area with u(0,0,t) = 0 (at boundaries for all time), initial displacement and initial velocity being f(x,y) and g(x,y). (15)

6. (a) Obtain x(t) and y(t) from

$$\frac{dx}{dt} = x + y$$

$$\frac{dy}{dt} = 3x - y$$

(b) Use Forbenius method to solve

$$8x^2 \frac{d^2y}{dx^2} + 10x\frac{dy}{dx} - (1 + x)y = 0$$ (5+10)

7. (a) Show that using the method of separating variables, the equation

$$\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial x^2}\right)$$

can be reduced to two ordinary differential equations.

(b) Derive the equation governing small transverse vibrations at any point 'x' and time 't', set in an elastic homogeneous string stretched to length 'L' and fixed at its two ends. Assuming disturbance was set at time $t = 0$, solve the wave equation assuming initial deflection and initial velocity to be given as $f(x)$ and $g(x)$ respectively. (3+12)