[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1106 **E** **Your Roll No................**

Unique Paper Code : 251606

Name of the Course : **B.Sc. (H) Electronics**

Name of the Paper : Engineering Mathematics [ELHT–604]

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **FIVE** questions in all.
3. Question No. **1** is compulsory.
4. **All** questions carry equal marks.
5. Attempt all parts of a question together.
6. Use of Scientific non-programmable calculators is allowed.

1. Attempt all the **five** parts :

(a) Classify the given differential equation by its kind, order, degree, linearity and homogeneity

$$D^4y + (D^3y)^3 + 2Dy = \cos^2 x$$

(b) Solve $(D^2 + 6D + 9)y = 5e^{3x}$.

(c) Show that x^2, $x^2 \ln(x)$ are linearly independent functions.

(d) Prove that $\beta(m, n) = \beta(n, m)$ for a Beta function.

(e) Solve :

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 3z \qquad (5\times3)$$

2. (a) Solve the differential equation for y(x)

$$D^2y + Dy + y = x^3$$

(b) Solve the given Cauchy-Euler equation

$$x^2d^2y/dx^2 - 3x\, dy/dx + 5y = x^2 \sin(\log x)$$

(c) If weight w = 16 kg, spring constant = 10 kg/ m, damping force is $2\dot{x}$, external force F(t) is 5 cos 2t, find the motion of the weight given $x(0) = \dot{x}(0) = 0$. Write the transient and steady state solutions. Also describe the nature of these solutions. (3+6+6)

3. (a) Find the power series solution of the linear oscillator in powers of x near x = 0.

$$\frac{d^2y}{dx^2} + \omega^2 y = 0$$

(b) For Bessel's function, prove that $J_{-n}(x) = (-1)^n J_n(x)$, where n is a positive integer. (9+6)

4. (a) Form the partial differential equation by eliminating the arbitrary function from the relation

$$z = x^n f\left(\frac{y}{x}\right)$$

where z is a dependent variable and x and y are the independent variables.

(b) Solve the partial differential equation :

$$(y+zx)\frac{\partial z}{\partial x}-(x+yz)\frac{\partial z}{\partial y}=x^2-y^2.$$

(c) Solve the following Partial Differential Equation by the method of separation of variables

$$u_x = 4xy, \quad u(0,y) = 8e^{-3y}$$ (3+6+6)

5. (a) Write down the partial differential equation for heat flow in a long thin homogeneous bar of length 'L', oriented along the x-direction only. With the two ends of the bar at temperature 0°C, the initial temperature of the bar at t=0 was given by the function f(x). Determine the solution u(x,t) under the given conditions.

(b) Derive the d'Alembert's solution of the one dimensional wave equation.

(10+5)

6. (a) Solve the simultaneous equations to obtain x(t) and y(t) :

$$\frac{dx}{dt} = x + 2y$$

$$\frac{dy}{dt} = 4x + 3y$$

Given x(0) = 6, y(0) = −2.

(b) Evaluate $\int_0^\infty \frac{x^{2m}}{1+x^{2n}}\,dx$ m, n > 0 and n > m (7+8)

7. (a) Form the partial differential equation by eliminating the arbitrary constants a and b from given equation z = axy + b.

(b) Considering the motion of a stretched elastic vibrating membrane in the x-y plane. Determine the solution $u_{m,n}(x, y, t)$ of the two dimensional wave equation for small transverse vibration, if 'p' is the mass of the membrane per unit area with u(0,0,t) = 0 (at boundaries for all time), initial displacement and initial velocity being f(x, y) and g(x, y). (5+10)