[This question paper contains 6 printed pages.]

5879 *Your Roll No.* ....

B.Sc. (Hons.) Geology / I Sem. **B**

Paper – GEHT–104 : Mathematics

(Admissions of 2010 and onwards)

Time : 3 hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **two** *questions from each Section.*
All *Sections carry equal marks.*
Use of scientific calculator is allowed.

SECTION I

1. (a) Draw the graph of the function

$$y = 4 - x^2$$

with domain $-2 \le x \le 2$. Also show that it is not a monotone function. (6½)

(b) Show that the sequence $<a_n>$ defined by

$$a_1 = 1,\ a_{n+1} = \sqrt{29_n}\ ,\ \ n \ge 1$$

is convergent. (6)

2. (a) Discuss the limiting behaviour of

$$f(x) = \frac{2x}{x^2 - 4}$$

when (i) $x \to 2$ (ii) $x \to -2$. (6)

(b) Find (i) $\lim\limits_{n\to\infty} \frac{\sqrt{4n^2 - 1}}{3n}$

(ii) $\lim\limits_{n\to\infty} \left(1 - \frac{3}{n}\right) \Big/ \left(\frac{1}{n} - 3\right)$ (6½)

3. (a) Find the following derivatives

(i) $\frac{d}{ds}\left(\ln\left(1 + \frac{1}{s}\right)\right)$

(ii) $\frac{d^2}{dx^2}\left(e^{-x^2/2}\right)$ (6)

(b) Show $\frac{d}{dx}(\tan hx) = \frac{1}{\cosh^2 x}$ by the first principle. (6½)

4. (a) Given $Q = (x^2 + y^2 + z^2)^{1/2}$. Then

(i) Find $\frac{\partial^2 Q}{\partial x^2} + \frac{\partial^2 Q}{\partial y^2} + \frac{\partial^2 Q}{\partial z^2}$

(ii) Is $\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial Q}{\partial z}\right)\right) = \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial Q}{\partial z}\right)\right)$?

Justify your answer. (6)

(b) If $Z = e^{x/y}\left(\sin\left(\frac{x}{y}\right)\right) + e^{y/x}\left(\cos\left(\frac{y}{x}\right)\right)$,

show that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0.$$ (6½)

SECTION II

5. (a) Find the integrals

(i) $\int_{x=-\frac{1}{2}}^{\frac{1}{2}} \sqrt{(1-2x)}\,dx$

(ii) $\int_{1}^{x}\left(\frac{au-1}{u+1}\right)dx$, a is a given constant. (6)

(b) Applying the Fundamental Theorem of Integral Calculus. Find the average value of $\frac{x-3}{\sin x}$, over [2, 3]. (6½)

6. (a) Find the area under the curve $y = \frac{1}{(x+5)^2}$, over the interval [3, 7]. (6½)

(b) Assume that, when a helical spring is extended moderately, Hooke's law is valid. It states that the amount of extension 's' is proportional to the extending force F, where 's' is measured in meters and F in Newtons. Find the energy W (to be measured in Joules), to extend the spring from $s = 0$ to $s = s_0$. (6)

7. (a) Classify the following differential equations w.r.t. the order, explicit-implicit, linear-nonlinear, homogeneous-nonhomogeneous, constant-variable coefficients :

(i) $u' = 5 - u$ (ii) $\frac{dy}{dx} = y \sin x$

(iii) $y = \frac{1}{\sin y'}$ (6)

(b) Find the general solution of the following system of differential equations :

$$\frac{dx}{dt} = 7x - 4y,$$

$$\frac{dy}{dt} = -9x + 7y.$$ (6½)

8. (a) Verify that $C(x,t) = t^{-1/2} \exp(-x^2/4Dt)$ is a particular solution of the diffusion equation

$$\frac{\partial c}{\partial t} = D.\frac{\partial^2 c}{\partial x^2}.$$ (6)

(b) Solve the differential equation

$$ny\frac{dy}{dx} = x^2 + y^2.$$ (6½)

SECTION III

9. (a) Find 'k' such that the system

$$kx + 3y - 2z = 0$$
$$(k-1)y + 7z = 0$$
$$(k+2)z = 0$$

has a nontrivial solution. (6½)

(b) Find eigenvalues and eigenvectors for

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 7 & 3 \\ 5 & 4 & 3 \end{bmatrix}$$ (6)

10. (a) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 4 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

Also, show that $(A^{-1})^{1} = (A^{1})^{-1}$. (6)

(b) With the help of elementary transformations find the rank of the matrix

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & -2 & 1 \\ 2 & 0 & -3 & 2 \\ 3 & 3 & -3 & 3 \end{bmatrix}$$ (6½)

11. (a) Show that

(i) $\frac{d}{dt}(\vec{u}\cdot\vec{v}) = \frac{d\vec{u}}{dt}\cdot\vec{v} + \vec{u}\cdot\frac{d\vec{v}}{dt}$ and

(ii) $\frac{d}{dt}(\vec{u}\times\vec{v}) = \frac{d\vec{u}}{dt}\times\vec{v} + \vec{u}\times\frac{d\vec{v}}{dt}$. (6)

(b) If $\vec{A} = (3x^2 - 6yz)\hat{i} + (2y + 3xz)\hat{j} + (1 - 4xyz^2)\hat{k}$, evaluate $\int_C \vec{A}\cdot d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the curve C given by $x = t,\ y = t^2,\ z = t^3$. (6½)

12. (a) Given $\frac{dy}{dx} = x^2 - y,\ y(0) = 1$, find y(0.1) using Runge Kutta method of 2nd order with $h = 0.1$. (6)

(b) Find the Laplacian of $\phi = f(r)$. Hence show that $\phi = \frac{1}{r}$ is a solution of Laplacian equation $\nabla^2\phi = 0$. (6½)