[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 6164 **D** **Your Roll No...............**

Unique Paper Code : 235182

Name of the Course : **B.Sc. (H) Geology**

Name of the Paper : Mathematics (GEHT-104)

Semester : I

Time : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **two** questions from each section.
3. **All** questions carry equal marks.
4. Use of scientific calculator is allowed.

SECTION I

1. (a) If $y = \sin(m \sin^{-1} x)$, then show that :

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 - m^2)y_n = 0.$$

Also, find $y_n(0)$. (6½)

(b) Use ε-definition, prove that $\lim_{n \to \infty} \frac{1}{n-1} = 0$ and check its convergence. (6)

2. (a) State three dimensional Laplacian equation and show that the function :

$f(x, y, z) = 2x^2 + 2y^2 - 4z^2$

satisfies it. (6)

(b) Evaluate the following integral.

(i) $\int x^2 \cos 6x dx$

(ii) $\int_{\frac{-1}{2}}^{\frac{1}{2}} \sqrt{(1-2x)}\,dx$ (6½)

3. (a) Compute the mixed second order partial derivatives of f, where

$$f(x,y) = x^2y + 3xy^2 + y^3$$

verify that those are same. (6)

(b) If $V = r^m$, where $r^2 = x^2 + y^2 + z^2$, show that

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = m(m+1)r^{m-2}$$ (6½)

4. (a) Verify Green's theorem in the plane for

$$\int_C (x^2 - 2xy)dx + (x^2y + 3)dy,$$

where C is the boundary of the region defined by $y^2 = 8x$ and $x = 2$. (6.5)

(b) Draw the graph of the function $y = 16 - x^2$ with domain $-4 \le x \le 4$. Also, show that it is not a monotone function. (6)

SECTION II

5. (a) Investigate the convergence of $\int_1^\infty \left(\frac{1}{\sqrt{x+1}} - \frac{1}{\sqrt{x}}\right) dx$. (6)

(b) Show that the differential equation

$$\left(x^2 + \cos y\right)\frac{dy}{dx} + 2xy = 0$$

is exact and solve it. (6½)

6. (a) Find the general solution of the following system of differential equations :

$$\frac{dx}{dt} = 7x - 4y,$$

$$\frac{dy}{dt} = -9x + 7y.$$ (6½)

(b) Find the area under the curve $y = \frac{1}{(x+5)^2}$ over the interval [3, 7]. (6)

7. (a) Find the area enclosed by the curve

$x = a\cos^3 t, \quad y = b\sin^3 t, \quad 0 \le t \le 2\pi$ (6½)

(b) Using double integral, find the volume in the first octant bounded by the co-ordinate planes and the plane $x + y + z = 1$. (6)

8. (a) Find the entire length of the cardioide $r = a(1 - \cos\theta)$. (6½)

(b) Evaluate $\int_0^1 \int_2^3 (x^2 + y)\,dydx$ and $\int_2^3 \int_0^1 (x^2 + y)\,dxdy$ and show that they are equal. (6)

SECTION III

9. (a) Find the eigenvalues and eigenvectors for

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 7 & 3 \\ 5 & 4 & 3 \end{bmatrix}$$ (6½)

(b) With the help of elementary transformations, find the rank of the matrix

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & -2 & 1 \\ 2 & 0 & -3 & 2 \\ 3 & 3 & -3 & 3 \end{bmatrix}$$ (6)

10. (a) Find 'k' such that the system

$$kx + 3y - 2z = 0$$

$$(k-1)y + 7z = 0$$

$$(k+2)z = 0$$

has a non-trivial solution. (6½)

(b) Compute the directional derivative of the function $f(x, y, z) = x^2yz + z^2y$ at the point (1, 2, 3) in the direction of the vector $\vec{a} = 2\hat{i} - 3\hat{j} + 7\hat{k}$. (6)

11. (a) Perform three iterations of Newton-Raphson method to find a root of $f(x) = x^4 - 18x^2 + 45$ with $x_0 = 1$. (6)

(b) Show that

(i) $\nabla\left(\frac{\vec{r}}{r}\right) = \frac{2}{r}$

(ii) $\nabla^2\left(\frac{1}{r}\right) = 0$ (6.5)

12. (a) Given $\frac{dy}{dx} = x^2 - y,\ y(0) = 1$, Find y(0.1) using Runge Kutta method of 2^{nd} order with step size h = 0.2. (6½)

(b) Which of the following transformations $T : R^2 \rightarrow R$ are linear ? Justify:

(i) $T(x,y) = 2x - y$

(ii) $T(x,y) = x + 1$ (6)

(100)