This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **841**

Unique Paper Code : **222101** **G**

Name of the Paper : **PHHT-101 : Mathematical Physics I**

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *Five* questions in all including Q. No. **1** which is compulsory.

1. Do any *five* of the following : 5×3=15

(*a*) Find the derivative of $f(x, y, z) = x^2 - xy^2 - xe^x$ at P(1, 1, 0) in the direction of :

$$\vec{A} = \hat{i} - 2\hat{j} + 2\hat{k}.$$

(*b*) Prove that :

$$\nabla^2\left(\frac{1}{r}\right) = 0.$$

(*c*) Prove :

$$\iiint \frac{dV}{r^2} = \iint \frac{\vec{r} \cdot \hat{n}}{r^2} ds.$$

P.T.O.

(*d*) Let u_1, u_2, u_3 be the orthogonal coordinates. Prove that :

$$\vec{\nabla} u_p = h_p^{-1}, \qquad p = 1, 2, 3.$$

(*e*) What is the period of the function :

$$f(t) = 3 \sin(2\pi t) + 2 \cos(\pi t)$$

(*f*) If $x = u^2 + 2$, $y = u + v$, $z = \omega^2 - u$, find the Jacobian :

$$J\left(\frac{x, y, z}{u, v, w}\right).$$

(*g*) Evaluate :

$$\sqrt{\Gamma\left(\frac{-5}{2}\right)}.$$

2\. (*a*) Prove that :

$$\vec{F} = \left(y^2 \cos x + z^3\right)\hat{i} + \left(2y \sin x - 4\right)\hat{j} + \left(3xz^2 + 2\right)\hat{k}$$

is a conservative force field. Find the scalar potential for $\vec{F}$. 5

(*b*) Prove the identity : 5

$$\vec{\nabla} \cdot \left(\vec{A} \times \vec{B}\right) = \vec{B} \cdot \left(\vec{\nabla} \times \vec{A}\right) - \vec{A} \cdot \left(\vec{\nabla} \times \vec{B}\right).$$

(*c*) Show that : 5

$$\vec{\nabla}\left(\frac{\vec{p} \cdot \vec{r}}{r^3}\right) = \frac{1}{r^3}\left(\vec{p} - 3\left(\vec{p} \cdot \hat{r}\right)\hat{r}\right).$$

3. (*a*) State and prove Gauss's Divergence theorem. 8

(*b*) Verify Green's theorem in the plane for :

$$\oint \left(3x^2 - 8y^2\right)dx + \left(4y - 6xy\right)dy,$$

where C is the boundary of the region defined by $y = \sqrt{x}, y = x^2$. 7

4. (*a*) Verify Stokes' theorem for :

$$\vec{V} = \left(3x - y\right)\hat{i} - 2yz^2\hat{j} - 2y^2z\hat{k},$$

where S is the upper half of the sphere $x^2 + y^2 + z^2 = 16$ and C is its boundary. 10

(*b*) Find the surface area of the plane $x + 2y + 2z = 12$ cut-off by the surfaces $x = 0$, $y = 0$ and $x^2 + y^2 = 16$. 5

5. (*a*) Derive an expression for divergence of a vector function $\vec{A}$ in orthogonal curvilinear coordinates system. 5

(*b*) Represent the vector $\vec{A} = x\hat{i} - 2z\hat{j} + y\hat{k}$, in cylindrical coordinates. Determine A_ρ, A_ϕ and A_z. 5

(*c*) Evaluate : 5

$$\iiint \left(x^2 + y^2 + z^2\right)dV,$$

where V is volume of the sphere :

$$x^2 + y^2 + z^2 = a^2.$$

6. (*a*) Expand as Fourier series, the function : 8

$$f(x) = \begin{cases} -k & -\pi < x < 0 \\ +k & 0 < x < \pi \end{cases}$$

Hence find the sum :

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \dots\dots\dots .$$

(*b*) If 7

$$f(x) = \begin{cases} x & 0 \le x \le \frac{\pi}{2} \\ \pi - x & \frac{\pi}{2} \le x \le \pi \end{cases}$$

Graph the function and express the function as cosine series.

7. (*a*) Prove that : 5

$$\beta(m, n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}.$$

(*b*) Evaluate : 5

$$\int_0^\infty \sqrt{x}\, e^{-x}\, dx.$$

(*c*) The radius of a cylinder is given as (2.0 ± 0.1) cm and height as (6.2 ± 0.2) cm. Find the volume of the cylinder and standard error in volume. 5