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Your Roll No.

510

Subsidiary for B.Sc. (Hons.)/I A
MATHEMATICS – Paper II
(Trigonometry and Algebra including Matrices and
Differential Equations)

Time : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any six questions.

All questions carry equal marks.

1. (a) If $x = \cos \alpha + i \sin \alpha$ and $y = \cos \beta + i \sin \beta$,
show that $\frac{x-y}{x+y} = i \tan \left(\frac{\alpha-\beta}{2} \right)$. 6
- (b) Find all the roots of the equation $z^7 - z = 0$. 6½

2. (a) Solve the equation $x^3 - 3x^2 + 4 = 0$, given
that two of its roots are equal. 6
- (b) Find the equation whose roots are the
squares of the roots of the equation
 $x^3 - x^2 - 8x + 6 = 0$. 6½

3. (a) Find a necessary condition for the roots of the equation $x^3 - px^2 + qx - r = 0$ to be in H.P. 6
- (b) If α, β, γ are roots of the equation $x^3 + px^2 + qx + r = 0$, find the values of
- (i) $\sum \frac{1}{\alpha}$ (ii) $\sum \frac{\alpha}{\beta}$ 6½
4. (a) Solve : $(1 + e^{x/y}) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy = 0$ 6
- (b) Solve : $y = 2px - xp^2$, where $p = \frac{dy}{dx}$. 6½
5. (a) Solve the differential equation $(D^3 + 1)y = \cos 2x$, where $D \equiv \frac{d}{dx}$. 6
- (b) Solve : $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = x^3$. 6½
6. (a) Test for convergence the series whose n^{th} term is $\left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$. 6
- (b) Discuss the convergence of the series $\sum_{n=1}^{\infty} (-1)^n \left(\frac{n+2}{2^n+5}\right)$. 6½

7. (a) For any positive real number r , show that the series $\sum_{n=1}^{\infty} \frac{r^n}{n!}$ is convergent. 6

(b) Test for convergence the series whose n^{th} term is $2^{-n-(-1)^n}$. 6½

8. (a) Find the rank of the matrix

$$\begin{pmatrix} 1 & 4 & 3 & 2 \\ 1 & 2 & 3 & 4 \\ 2 & 6 & 7 & 5 \end{pmatrix} \quad 6$$

(b) Solve completely the system of homogeneous equations :

$$2x - 3y + z = 0$$

$$x + 2y - 3z = 0$$

$$4x - y - 2z = 0 \quad 6½$$

9. (a) Is the following system of equations consistent ?

$$x - 3y + z = -1$$

$$2x + y - 4z = -1$$

$$6x - 7y + 8z = 7$$

Justify your answer. 6

(b) If a, b, c are distinct real numbers satisfying $a + b + c = 0$, show that the rank of the

matrix $\begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{pmatrix}$ is two. 6½