This question paper contains 4 printed pages.]

Your Roll No. .....

509

Subsidiary for B.Sc. Honours/I A

MATHEMATICS – Paper I

(Differential Calculus and Integral Calculus)

Time : 3 Hours **Maximum Marks : 75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **six** questions in all, selecting at least **two** questions from each Section.

All questions carry equal marks.

Parts of a question should be attempted together.

SECTION – I

1. (a) Verify Lagrange's Mean Value Theorem for the function

$$f(x) = x\,(x-1)\,(x-2) \text{ in } \left[0, \frac{1}{2}\right]$$

(b) Evaluate $\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{\sin x}\right)$.

2. (a) If $y = e^{m \sin^{-1} x}$, then show that

$(1 - x^2)y_{n+2} - (2n + 1)x\, y_{n+1} - (n^2 + m^2)y_n = 0.$

Also find $y_n(0)$.

(b) Prove that the points of the curve

$$y^2 = 4a\left(x + a \sin \frac{x}{a}\right)$$

at which the tangent is parallel to the axis of x, lie on a parabola.

3. (a) Show that for the curve $x = a \cos \theta (1 + \sin \theta)$, $y = a \sin \theta (1 + \cos \theta)$, the radius of curvature is, a, at the point for which the value of the parameter θ is $-\frac{\pi}{4}$.

(b) If $u = \cot^{-1}\left(\frac{x + y}{\sqrt{x} + \sqrt{y}}\right)$, then prove that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + \frac{1}{4} \sin 2u = 0.$$

4. (a) Trace the curve $y^2 x = a^2(a - x)$.

(b) Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius a is $\frac{2a}{\sqrt{3}}$.

5. (a) Find the position and nature of the double points of the curve

$x^4 - 2ax^2y - axy^2 + a^2x^2 = 0$

(b) Find the asymptotes of the curve

$x^3 + x^2y - xy^2 - y^3 + 2xy + 2y^2 - 3x + y = 0$

SECTION – II

6. (a) If $I(m, n) = \int_0^{\pi/2} \cos^m x \cos nx \, dx$, then prove that

(i) $I(m, n) = \frac{m}{m+n} I(m-1, n-1)$

(ii) $I(m, n) = \frac{m(m-1)}{m^2 - n^2} I(m-2, n)$

(b) Obtain a reduction formula for $\int \sec^n x \, dx$ and deduce the value of $\int (a^2 + x^2)^{5/2} dx$.

7. Evaluate any **two** of the following :

(i) $\int_0^{\pi/2} \log \sin x \, dx$

(ii) $\int \frac{dx}{(x-1)^2 (x^2+4)}$

(iii) $\int x \sqrt{1 + x - x^2} \, dx$

8. (a) Find the volume of the solid obtained by the revolution of the curve $y^2(2a - x) = x^3$ about its asymptote.

(b) Find the surface of the solid generated by revolving the curve $r^2 = a^2 \cos 2\theta$ about the initial line.

9. (a) Find the area between the curve

$$x(x^2 + y^2) = a(x^2 - y^2)$$

and its asymptote. Also, find the area of its loop.

(b) Find the length of the loop of the curve $3ay^2 = x(x - a)^2$.

10. (a) Evaluate $\int\int \sqrt{4x^2 - y^2}\, dx\, dy$ over the triangle formed by the straight lines $y = 0, x = 1, y = x$.

(b) Evaluate $\int\int\int (x + y + z + 1)^4\, dx\, dy\, dz$ over the region defined by $x \geq 0$, $y \geq 0$, $z \geq 0, x + y + z \leq 1$.
