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Your Roll No. ....

2130

B.Sc. (Hons.)/I **C**

MATHEMATICS—Paper I

(Calculus)

(Admissions of 2009 and 2010)

Time : 3 Hours *Maximum Marks : 75*

(*Write your Roll No. on the top immediately on receipt of this question paper.*)

All the sections are compulsory.

Use of non-programmable scientific calculator is allowed.

Marks of each part are indicated.

Section I

Attempt any *three* questions from Section I

1. Find the constants A, B and C that guarantee that the function :

$$f(x) = Ax^3 + Bx^2 + C$$

will have relative extremum at (2, 11) and inflection point at (1, 5). 4

P.T.O.

2. Find the nth derivative of 4

$$y = \sin^2 x \cos x.$$

3. Suppose you have a rare book whose value t years from now is modelled as $300\, e^{\sqrt{3t}}$, if the prevailing rate of interest remains constant at 8% compounded continuously when will be the most advantageous time to sell the book : 4

$$(P(t) = v(t)e^{-rt})$$

4. Find A so that :

$$\lim \left(\frac{x+A}{x-2A}\right)^x = 5$$

as x tends to ∞. 4

Section II

Attempt any *three* questions from Section II.

5. The velocity of a particle moving in space is :

$$V(t) = e^t\mathbf{i} + t^2\mathbf{j} + \cos 2t\mathbf{k}.$$

Find the particle's position as a function of t if the position at time $t = 0$ is : 4

$$\mathbf{R}(0) = 2\mathbf{i} + \mathbf{j} - \mathbf{k}.$$

6. A shell fired from the ground level at an angle 45 degree hits the ground 2000 m away. What is muzzle speed of the shell ? 4

7. Prove that orthogonality of a function of constant length and its derivative. 4

8. Find an equation of the line which passes through the point Q(2, −1, 3) and is orthogonal to the plane :

$$3x - 7y + 5z + 55 = 0$$

where does the line intersect the plane. 4

Section III

Attempt any *two* questions from Section III.

9. Identify and sketch the curve : 6

$$x^2 + 9y^2 + 2x - 18y + 1 = 0.$$

10. Trace the conic :

$$x^2 - 3xy + y^2 + 10x - 10y + 21 = 0$$

by rotating the co-ordinate axis to remove the xy term. 6

11. Find an equation of the parabolic trace of :

$$z = y^2 - x^2 \text{ in } x = 2.$$

Find the vertices and foci of the trace. Also describe the orientation of the focal axis relative to the co-ordinate axis. 6

12. Find the distances from the pole to the vertices and find the equation of the ellipse in the rectangular co-ordinates for : 6

$$r = \frac{6}{2 + \sin\theta}.$$

Section IV

Attempt any *three* questions from Section IV.

13. Show that :

$$\text{Lim } \frac{x^2 y}{x^4 + y^2} \text{ as } (x, y)$$

tends to (0, 0) does not exist.

14. Find the absolute extrema of the function :

$$f(x) = e^{x^2 - y^2}$$

over the disk $x^2 + y^2 \leq 1$.

15. Let

$$f(x, y, z) = zyz$$

and let **u** be a unit vector perpendicular to both

$$\mathbf{v} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \mathbf{w} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}.$$

Find the directional derivative of f at P(1, −1, 2) in the direction of **u**. 4

16. A juice can is 12 cm tall and has radius of 3 cm. A manufacturer is planning to reduce the height of the can by 0.2 cm and radius by 0.3 cm. Use a total differential to estimate the percentage decrease in volume of the new can. 4

Section V

Attempt any *three* questions from Section V

17. Sketch the graph of

$$r^2 = 4\cos 2\theta$$

in polar co-ordinates. 4

18. Find the curvature of the ellipse :

$$\mathbf{r} = 2\cos t\ \mathbf{i} + 3\sin t\ \mathbf{j}\ (0 \le t \le 2\pi)$$

at the end points of the major and minor axis. 4

19. Find the equations of the surfaces

$$z = x^2 + y^2$$

in cylindrical co-ordinates and spherical co-ordinates. 4

20. A rectangular plot of land is to be fenced off so that the area enclosed will be 400 ft^2. Let L be the length of the fencing needed and x the length of one side of the rectangle. Show that :

$$L = 2x + \frac{800}{x}$$

for $x > 0$ and the sketch the graph of L versus x for $x > 0$. 4

Section VI

Attempt any *three* questions from Section VI.

21. Show that : 5

$$\int_0^{\frac{\pi}{3}} \cos^4 3\theta \sin^2 6\theta d\theta = \frac{5}{96}\pi.$$

22. Find the volume of the solid generated when the region R under $y = x^2$ over the interval [0, 2] is revolved about x-axis. 5

23. Find the area of the surface generated by revolving the curve : 5

$$y = x^3, 0 < x \leq \frac{1}{2}$$

about the x-axis.

24. Find the arc length of the curve : 5

$$y = x^{\frac{3}{2}} \text{ from } (1, 1) \text{ to } (2, 2\sqrt{2}).$$