[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1877 **C** **Roll No............**

Unique Paper Code : 235203

Name of the Course : B.Sc. (Hons.) Mathematics

Name of the Paper : MAHT-202 : Analysis – II

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **three** parts from each question.
3. **All** questions are compulsory.

1. (a) Use the $\epsilon - \delta$ definition of the limit to find

$$\lim_{x \to c} \frac{1}{x}, \quad c > 0$$

(b) By using sequential criteria for limits, prove that $\lim_{x \to 0} \operatorname{sgn} \sin\left(\frac{1}{x}\right), (x \neq 0)$ does not exist.

(c) Let f,g be defined on $A \subseteq R$ to R, and let c be a cluster point of A. Suppose that f is bounded on a neighbourhood of c and $\lim_{x \to c} g = 0$. Prove that $\lim_{x \to c} fg = 0$.

(d) Prove that :

(i) $\lim_{x \to 0} \frac{1}{\sqrt{|x|}} = +\infty, \quad x \neq 0$

(ii) $\lim_{x \to -\infty} \frac{1}{x^2} = 0, \quad x \neq 0$ (5,5,5,5)

2. (a) Prove that a function $f : A \to R$ is continuous at the point $c \in A$ if and only if for every sequence (x_n) in A that converges to c, the sequence $(f(x_n))$ converges to f(c).

(b) Determine the points of continuity of the function x[x], where $x \to [x]$ denotes the greatest integer function.

(c) Let f and g be continuous from R to R and suppose that f(r) = g(r) for all rational numbers r. Show that $f(x) = g(x), \forall x \in R$.

(d) State Bolzano's Intermediate Value Theorem. Show that x = cos (x) for some x in $\left]0, \frac{\pi}{2}\right[$. (5,5,5,5)

3. (a) Define uniformly continuous function on a set $A \subseteq R$. Show that if f and g are uniformly continuous on A and if they are both bounded on A, then their product fg is uniformly continuous on A.

(b) State non uniform continuity criteria. Show that the function $f(x) = x^2$ is not uniformly continuous on the set $A = [0, \infty[$

(c) Let $f(x) = |x| + |x - 1|$. Check the differentiability of f at x = 0 and x = 1.

(d) Prove that if f is differentiable at a and g is differentiable at f(a), then the composite function gof is differentiable at a and $(gof)'(a) = g'(f(a)).f'(a)$.

(5,5,5,5)

4. (a) Let c be an interior point of the interval I at which $f: I \to R$ has a relative extremum. If the derivative of f at c exists, then show that $f'(c) = 0$.

(b) Show that $x < \tan x \ \forall\ x \in \left]0, \frac{\pi}{2}\right[$ and deduce that $\frac{x}{\sin x}$ is a strictly increasing function on $\left]0, \frac{\pi}{2}\right[$.

(c) Let $a_1, a_2, \ldots, a_n$ be real numbers and let f be defined on R by

$$f(x) = \sum_{i=1}^{n}(a_i - x)^2$$

Find the unique point of relative minimum for f.

(d) Suppose that $f : [0,2] \to R$ is continuous on [0,2] and differentiable on]0,2[and that $f(0) = 0$, $f(1) = 1$, $f(2) = 1$.

(i) Show that there exists $c_1 \in]0,1[$ such that $f'(c_1) = 1$.

(ii) Show that there exists $c_2 \in]1,2[$ such that $f'(c_2) = 0$.

(iii) Show that there exists $c \in]0,2[$ such that $f'(c) = \frac{1}{3}$. (5,5,5,5)

5. (a) State and prove Taylor's Theorem.

(b) Obtain Maclaurin's series expansion for the function cos x.

(c) Show that sin x lies between $x - \frac{x^3}{6}$ and x, $\forall\ x \in R$.

(d) Define a convex function on an interval $I \subseteq R$. Check which of the following functions are convex :

(i) $|x|, x \in [-1, 1]$

(ii) $x + \sin x, x \in [0, \pi]$ (5,5,5,5)