[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1878 **C** **Roll No............**

Unique Paper Code : 235204

Name of the Course : B.Sc. (Hons.) Mathematics

Name of the Paper : Probability & Statistics – MAHT 203

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. In all there are **six** questions.
3. Question No. **1** is compulsory and it contains five parts of **3** marks each.
4. In Question No. **2** to **6**, attempt any **two** parts from **three** parts. Each part carries **6** marks.
5. Use of scientific calculator is allowed.

1. (i) If the sample space is $\mathcal{C} = \{c : 0 < c < \infty\}$. Let $C \subseteq \mathcal{C}$ is a set for which the integral

$$\int_C e^{-|x|}\,dx$$

exists, show that this set function is not a probability set function. What constant do we multiply the integrand by to make it a probability set function ?

(ii) Let X have the pdf

$$f(x) = \begin{cases} 2(1-x) & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Then, find $E(6X + 3X^2)$.

(iii) Let $f(x_1, x_2) = \begin{cases} 6x_1^2 x_2 & 0 < x_1 < 1, 0 < x_2 < 1 \\ 0 & \text{elsewhere} \end{cases}$

be the pdf of two random variables X_1 and X_2 of the continuous type. Find the value of $P\left(0 < X_1 < \frac{3}{4}, \frac{1}{3} < X_2 < 2\right)$.

(iv) Define probability set function. If C_1 and C_2 are events such that $C_1 \subseteq C_2$, then prove that $P(C_1) \leq P(C_2)$.

(v) If the probability is 0.75 that an applicant for a driver's license will pass the road test on any given try, what is the probability that an applicant will finally pass the test on the fourth try ?

2. (a) Let $\{C_n\}$ be an increasing sequence of events. Then, show that

$$\lim_{n \to \infty} P(C_n) = P\left(\lim_{n \to \infty} C_n\right) = P\left(\bigcup_{n=1}^{\infty} C_n\right)$$

(b) Find the probability of getting five heads and seven tails in 12 flips of a balanced coin.

(c) Show that if a random variable has a uniform density with parameters α and β, the r^{th} moment about the mean equals

(i) 0, when r is odd

(ii) $\frac{1}{r+1}\left(\frac{\beta - \alpha}{2}\right)$, when r is even

3. (a) Let X be a random variable having negative binomial distribution with parameters k and θ. Find the variance of X.

(b) Find the moment-generating function of the Poisson distribution and hence find its mean and variance.

(c) If X is a random variable having a binomial distribution with the parameters n and θ, then the moment-generating function of

$$Z = \frac{X - n\theta}{\sqrt{n\theta(1-\theta)}}$$

approaches that of the standard normal distribution when $n \to \infty$.

4. (a) Let 13 cards be taken, at random and without replacement, from an ordinary deck of playing cards. If X is the number of spades in these 13 cards, find the pmf of X. If, in addition, Y is the number of hearts in these 13 cards, find the probability $P(X = 2, Y = 5)$. What is the joint pmf of X and Y ?

(b) Let X_1 and X_2 have the joint pdf

$$f(x_1, x_2) = \begin{cases} 2 & 0 < x_1 < x_2 < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the conditional pdf of X_1 given $X_2 = x_2$, $0 < x_2 < 1$. Find conditional mean and conditional variance of X_1, given $X_2 = x_2$.

Evaluate $P\left(0 < X_1 < \frac{1}{2} \mid X_2 = \frac{3}{4}\right)$ and $P\left(0 < X_1 < \frac{1}{2}\right)$.

(c) Let X be a random variable with cumulative distribution function $F_X(X)$, then prove that

(i) F_X is an increasing function.

(ii) F_X is everywhere continuous to the right.

5. (a) If (X, Y) has a bivariate normal distribution, find the marginal distribution of X and Y. Under what conditions X and Y are independent ?

(b) If the regression of Y on X is linear, then show that

$$\mu_{Y|x} = \mu_2 + \rho\frac{\sigma_2}{\sigma_1}(x - \mu_1)$$

(c) Let X and Y have the joint pdf given by

$$f(x, y) = \begin{cases} 6y & 0 < y < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Then show that $E[E(Y|X)] = E(Y)$ and $var[E(Y|X)] \le var(Y)$.

P.T.O.

6. (a) State and prove Chebyshev's Theorem.

(b) Suppose that whether or not it rains today depends on previous weather conditions through the last two days. Specifically, suppose that if it rained for the past two days, then it will rain tomorrow with probability 0.7; if it rained today but not yesterday, then it will rain tomorrow with the probability 0.5; if it rained yesterday but not today, then it will rain tomorrow with probability 0.4; if it has not rained in the past two days, then it will rain tomorrow with probability 0.2. Express this model as a Markov chain and find its transition probability matrix. Given that it rained on Monday and Tuesday, what is the probability that it will rain on Thursday ?

(c) Let X be the number of times that a fair coin, flipped 40 times, lands heads. Find the probability that $X = 20$. Use the normal approximation and then compare it to the exact solution.