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Your Roll No. ....

511

Subsidiary for B.Sc. Honours / II A

MATHEMATICS – Paper III

(Analytic Geometry of Two Dimensions and Vectors)

Time : 3 Hours **Maximum Marks : 75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any **six** questions.

All questions carry equal marks.

1. (a) Find λ such that the equation $12x^2 - 10xy + 2y^2 + 11x - 5y + \lambda = 0$ may represent a pair of straight lines. **6**

(b) Prove that the angle between the lines joining the origin to points of intersection of the line $y = 3x + 2$ with the curve $x^2 + 2xy + 3y^2 + 4x + 8y = 11$ is $\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)$. **6½**

2. (a) Find the equation to the circle which cuts orthogonally each of the three circles

$x^2 + y^2 + 2x + 17y + 4 = 0,$

$x^2 + y^2 + 7x + 6y + 11 = 0,$

$x^2 + y^2 - x + 22y + 3 = 0$ **6**

(b) Find the limiting points of the coaxal system of circles of which two members are $x^2 + y^2 - 6x + 12y + 5 = 0$ and $3(x^2 + y^2) + 10x - 20y + 15 = 0$. 6½

3. (a) Prove that the portion of a tangent to parabola cut off between the directrix and the curve subtends a right angle at the focus. 6

(b) Prove that through any point 3 normals can be drawn to the parabola and the algebraic sum of the ordinates of the feet of the three normals from any given point is zero. 6½

4. (a) Show that locus of the poles of normal chords to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is the curve $\frac{a^6}{x^2} + \frac{b^6}{y^2} = (a^2 - b^2)^2$. 6

(b) If P and D are extremities of conjugate diameters of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, show that the tangents at P and D meet on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$. 6½

5. (a) Show that the line $lx + my = n$ will touch the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ if $a^2l^2 - b^2m^2 = n^2$. 6

(b) Show that, if two diameters be conjugate with respect to a given hyperbola, they will also be conjugate with respect to the conjugate hyperbola. 6½

6. (a) Show that $\frac{l}{r} = 1 + e\cos\theta$ and $\frac{l}{r} = -1 + e\cos\theta$ represent same conic. 6

(b) Prove that the condition that the line $\frac{l}{r} = A\cos\theta + B\sin\theta$ may touch the conic $\frac{l}{r} = 1 + e\cos\theta$ is $(A - e)^2 + B^2 = 1$. 6½

7. Trace the conic $4x^2 - 4xy + y^2 - 8x - 6y + 5 = 0$.

12½

8. (a) Find the area of the parallelogram whose adjacent sides are respectively $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = -3\hat{i} - 2\hat{j} + \hat{k}$. 6

(b) If $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$, calculate $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$. 6½

9. (a) A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2\cos 3t$, $z = 2\sin 3t$. Find the velocity and acceleration of the particle at time $t = 0$. 6

(b) If $\vec{a} = t^2\hat{i} - t\hat{j} + (2t + 1)\hat{k}$ and $\vec{b} = (2t + 3)\hat{i} + \hat{j} - t\hat{k}$, find $\frac{d}{dt}(\vec{a} \times \vec{b})$. 6½

10. (a) If $\vec{R}(u) = (u - u^2)\hat{i} + 2u^3\hat{j} - 3\hat{k}$, find $\int \vec{R}(u)\, du$. 6

(b) Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$

and C is the arc of the curve

$\vec{r} = t\hat{i} + t^2\hat{j} + t^3\hat{k}$ from $t = 0$ to $t = 1$. 6½