[This question paper contains 4 printed pages.]

5112 *Your Roll No.* .....

Subsidiary for B.Sc. Honours/II **J**

MATHEMATICS – Paper IV (*i*)

(Mechanics)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any **six** *questions.*
All *questions carry equal marks.*
Symbols used have their usual meanings.

1. (a) A particle of mass m at rest at t = 0 is subject to a force $F(t) = F_0 \sin^2 wt$. (a) Sketch the form you expect for v(t) and x(t), for several periods of oscillation fo the force. (b) Find v(t) and x(t) and compare with your sketch. (6)

   (b) A boat is slowed by a frictional force F(v). Its velocity decreases according to the formula

   $$v = C(t - t_1)^2,$$

   where C is a constant and t_1 is the time at which it stops. Find the force F(v). (6½)

P.T.O.

2. (a) A particle of mass m is subject to a force

$$F = -kx + \frac{kx^3}{a^2}$$

where k, a are constants. Find V(x) and discuss the kinds of motion which can occure. (6½)

(b) A projectile is fired vertically upward with an initial velocity v_0. Find its motion, assuming a frictional drag proportional to the square of velocity. (6)

3. (a) Discuss the types of motion that can occure for a central force $F(r) = -\frac{K}{r^2} + \frac{K^1}{r^3}$. Assume that $K > 0$, and consider both signs for K^1. (6½)

(b) A particle is projected vertically upward with a velocity 4. Find its velocity at any height h. (6)

4. (a) A particle of mass m moves according to the eqns.

$$x = x_0 + at^2, \quad y = bt^3, \quad z = ct.$$

Find the angular momentum L at any time t. Find the force F and from it the torque N acting on the particle. (6½)

(b) Find the r– and θ– components of $\frac{da}{dt}$ in plane polar co-ordinates, where 'a' is the acceleration of a particle. (6)

5. (a) Calculate the energy loss – Q for a headon collision between a particle of mass m_1, velocity v_1, with a particle of mass m_2 at rest, if the coefficient of restitution is e. (6)

(b) A particle of mass m_1, momentum p_1, collides with a particle of mass m_2 at rest. A reaction occures from which two particles of masses m_3 and m_4 results, which leave the collision at angles θ_3 and θ_4 with the original path of m_1. Find the energy Q produced in the reaction in terms of the masses, the angles and p_1. (6½)

6. (a) The balance wheel of a watch consists of a ring of mass M, radius a with spokes of negligible mass. The hairspring exerts a restoring torque $N_z = -k\theta$. Find the motion if the balance wheel is rotated through an angle θ_0 and released. (6½)

(b) Find the centre of mass of a wire bent in to a semicircle of radius a. (6)

7. (a) Find the formula for radius of gyration of a uniform rod of length l about an axis through one end making an angle α with the rod. (6)

(b) A cable 20ft. long is suspended between two points A and B, 15ft. apart. The line AB makes an angle

of 30° with the horizonal (B higher). A weight of 2000 lb is hung from a point C 8ft. from the end of the cable at A. Find the position of point C, and the tensions in the cable, if the cable does not stretch. (6½)

8 (a) A mass αM is located at $x = a$, $y = 0$ and a second mass $(1-\alpha)M$ is located at $x = 0$, $y = b$, where $0 < \alpha < 1$. Find the co-ordinates x, y of the centre of gravity of the two masses relative to the origin. Show that formulas for x, y have the proper limits when $\alpha \to 0$ or $b \to \infty$. (6½)

(b) Use Kepler's third law to show that the forces on the planents are proportional to their masses. (6)

9. (a) Find the boundary condition and the normal modes of vibration of a streched string of length l terminated at the end $x = h$ by a ring of negligible mass which slides without friction on a verticle rod. (6)

(b) A long streched string of tension τ and density σ_1 is tied at $x = 0$ to a string of density σ_2. If the mass of the knot is negligible, show that u and $\partial u/\partial x$ must be same on both sides of the knot. (6½)