[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1591 **C** **Roll No............**

Unique Paper Code : 235463

Name of the Course : B.Sc. (Hons.)

Name of the Paper : Mathematics II (Analysis and Statistics) PHHT-413

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt as per directions.
3. Students can use standard normal tables.

SECTION I

Do any **two** questions.

1. (a) If f_n is continuous on an interval $I \subseteq \mathbf{R}$ to $\mathbf{R}$ for each $n \in N$ and if $\sum_{n=1}^{\infty} f_n$ converges to f uniformly on I, then prove that f is continuous on I.

   (b) Show that the sequence $<f_n>$ where $f_n(x) = x^n$ is uniformly convergent in $[0,k]$, $k<1$ and only pointwise convergent in $[0,1]$. (6,6½)

2. (a) Show that the sequence $<f_n>$ where

$$f_n(x) = \begin{cases} n^2 x, & 0 \le x < \frac{1}{n} \\ -n^2 x + 2n, & \frac{1}{n} \le x < \frac{2}{n} \\ 0, & \frac{2}{n} \le x \le 1 \end{cases}$$

is not uniformly convergent on $[0,1]$.

(b) Show that the series $\sum_{n=1}^{\infty} \frac{x}{n(n+1)}$ is uniformly convergent in $[0,k]$, where k is any positive real number but it does not converge uniformly in $[0,\infty)$. (6,6½)

3. (a) Prove that a power series $\sum_{n=0}^{\infty} a_n x^n$ having radius of convergence $R > 0$ converges uniformly and absolutely in $[-R+\varepsilon, R-\varepsilon]$ for each $\varepsilon > 0$.

(b) Show that

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \quad -1 < x \le 1$$

and deduce that

$$\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$$ (6,6½)

SECTION II

4. Do any **three** parts :

(a) Show that $\int_0^\infty \left(\frac{1}{e^x - 1} - \frac{1}{x} + \frac{1}{2}\right)\frac{e^{-\lambda x}}{x}dx$, $\lambda > 0$ converges.

(b) Examine the convergence of $\int_0^\infty \frac{\sin x(1-\cos x)}{x^n}dx$.

(c) Show that

$$\int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}}dx = \beta(m,n)$$

(d) Show that

$$\int_0^\infty xe^{-x^8}dx \times \int_0^\infty x^2e^{-x^4}dx = \frac{\pi}{16\sqrt{2}}$$

(e) Show that

$$\int_0^{\pi/2} \log\left(a^2\cos^2\theta + b^2\sin^2\theta\right)d\theta = \pi\log\left(\frac{a+b}{2}\right)$$ (5,5,5,5,5)

SECTION III

5. Do any **one** part :

(a) The distribution function of a random variable X is given by

$$F(x) = \begin{cases} 1-(1+x)e^{-x}, & \text{for } x > 0 \\ 0, & \text{for } x \le 0 \end{cases}$$

Find $P(X \le 2)$, $P(1 < X < 3)$ and $P(X > 4)$.

(b) If the probability density of X is given by

$$f(x) = \begin{cases} \frac{x}{2}, & \text{for } 0 < x \le 1 \\ \frac{1}{2}, & \text{for } 1 < x \le 2 \\ \frac{3-x}{2}, & \text{for } 2 < x < 3 \end{cases}$$

and $f(x) = 0$ elsewhere, find the expected value of $g(X) = X^2 - 5X + 3$.

(5,5)

6. Do any **three** parts :

(a) Derive Poisson distribution as a limiting form of Binomial distribution.

(b) (i) Show that there is no value of k for which

$f(x,y) = ky(2y-x)$; for $x = 0,3$; $y = 0,1,2$

can serve as the joint probability distribution of two random variables.

(ii) Find the probability of getting five heads and seven tails in 12 flips of a balanced coin.

(c) Suppose that the amount of cosmic radiation to which a person is exposed when flying by jet across the United States is a random variable having normal distribution with a mean of 4.35 mrem and standard deviation of

0.59 mrem. What is the probability that a person will be exposed to more than 5.20 mrem of cosmic radiation on such a flight ?

(d) If the joint probability density of X and Y is given by

$$f(x,y) = \begin{cases} \frac{1}{4}(2x+y), & \text{for } 0 < x < 1 \text{ and } 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

find the marginal densities of X and Y and the conditional density of X given Y = 1. (5,5,5,5)

7. Do any **three** parts :

(a) In two large populations there are 35% and 30% of brown eyed people. Is the difference likely to be revealed by simple samples of 1500 and 1000 respectively from the two populations ?

(b) A drug was administered to 10 patients and the increments in their blood pressure were recorded as :

6, 3, 2, 4, −3, 4, 6, 0, 0 and 2

Can it be concluded that the drug has no effect on change in blood pressure ? ($t_{0.05}$ at 9 d.f. = 2.26)

(c) Two independent samples of 8 and 7 items respectively had the following values of the variables :

Sample I : 9 11 13 11 15 9 12 14

Sample II : 10 12 10 14 9 8 10

Do the estimates of population variance differ significantly ?

(Given that for 7 and 6 d.f. the value of F at 5% level of significance is 4.21)

(d) A die is tossed 120 times and each outcome is recorded as follows :

Faces	1	2	3	4	5	6
Frequency	20	22	17	18	19	24

Is the distribution of outcomes uniform ? ($\chi^2_{0.05}(5) = 11.07$) (5,5,5,5)