[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1880 **C** **Roll No............**

Unique Paper Code : 235401

Name of the Course : B.Sc. (Hons.) Mathematics

Name of the Paper : MAHT 401 : Differential Equations and Mathematical Modelling - II

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.

SECTION – I

1. Attempt any **two** out of the following :

   (a) Find the solution of the equation

   $u(x+y)\, u_x + u\,(x-y)\, u_y = x^2 + y^2$

   with the Cauchy data $u = 0$ on $y = 2x$. (7½)

   (b) Apply $v = \ln u$ and then $v(x,y) = f(x) + g(y)$ to solve

   $x^2(u_x)^2 + y^2(u_y)^2 = u^2$. (7½)

   (c) Find the solution of the initial - value system

   $u_t + 2uu_x + v = x, \quad v_t - cv_x = 0$

   with $u(x,0) = x$ and $v(x,0) = x$. (7½)

SECTION – II

2. Attempt any **one** out of the following :

(a) Derive the *damped wave equation* of a string

$$u_{tt} + a u_t = c^2 u_{xx},$$

where the damping force is proportional to the velocity and a is a constant. Considering a restoring force proportional to the displacement of a string, show that the resulting equation is

$u_{tt} + a u_t + b u = c^2 u_{xx}$, where b is a constant. (6)

(b) Obtain the general solution of the equation

$x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} + xy u_x + y^2 u_y = 0.$ (6)

3. Attempt any **two** out of the following :

(a) Obtain the general solution of the equation

$3 u_{xx} + 4 u_{xy} - \frac{3}{4} u_{yy} = 0.$ (6)

(b) Transform the equation to the form $v_{\xi\eta} = cv$, c = constant,

$$u_{xx} - u_{yy} + 3u_x - 2u_y + u = 0,$$

by introducing the new variables $v = u e^{(a\xi + b\eta)}$, where a and b are undetermined coefficients. (6)

(c) Transform the equation

$$u_{xx} + y u_{yy} + \sin(x+y) = 0$$

into the canonical form. Use the canonical form to find the general solution. (6)

SECTION – III

4. Attempt any **three** out of the following :

(a) Determine the solution of the initial-value problem for the semi-infinite vibrating string with a fixed end

$u_{tt} = c^2 u_{xx}$, $\quad 0 < x < \infty,\ t > 0$

$u(x,0) = f(x)$, $\quad 0 \le x < \infty$

$u_t(x,0) = g(x)$, $\quad 0 \le x < \infty$

$u(0,t) = 0$, $\quad 0 \le t < \infty$. (7)

(b) Determine the solution of the initial-value problem

$u_{tt} - c^2 u_{xx} = x$, $\quad u(x,0) = 0$, $\quad u_t(x,0) = 3$. (7)

(c) Find the solution of the initial boundary-value problem

$u_{tt} = u_{xx}$, $\quad 0 < x < 2,\ t > 0$

$u(x,0) = \sin(\pi x/2)$, $\quad 0 \le x \le 2$

$u_t(x,0) = 0$, $\quad 0 \le x \le 2$

$u(0,t) = 0$, $\quad u(2,t) = 0$, $t \ge 0$ (7)

(d) Solve the characteristic initial-value problem

$x\, u_{xx} - x^3 u_{yy} - u_x = 0$, $x \ne 0$,

$u(x,y) = f(y)$ on $y - x^2/2 = 0$ for $0 \le y \le 2$,

$u(x,y) = g(y)$ on $y + x^2/2 = 4$ for $2 \le y \le 4$, where $f(2) = g(2)$. (7)

SECTION – IV

5. Attempt any **three** out of the following :

(a) Determine the solution of the initial boundary-value problem by the method of separation of variables

$u_{tt} = c^2 u_{xx}$, $\quad 0 < x < \pi,\ t > 0$

$u(x,0) = 0,\ u_t(x,0) = 8\sin^2 x$, $\quad 0 \le x \le \pi$,

$u(0,t) = u(\pi,t) = 0$, $\quad t > 0$ (7)

(b) Prove the uniqueness of the solution of the initial boundary - value problem

$u_{tt} = c^2 u_{xx}$, $\quad 0 < x < \pi,\ t > 0$

$u(x,0) = f(x),\quad u_t(x,0) = g(x)$, $\quad 0 \le x \le \pi$,

$u_x(0,t) = u_x(\pi,t) = 0$, $\quad t > 0$. (7)

(c) Solve the problem

$u_{tt} = c^2 u_{xx} + x^2,\ 0 < x < 1,\ t > 0$

$u(x,0) = x,\ u_t(x,0) = 0,\ 0 \le x \le 1$,

$u(0,t) = 0,\ u(1,t) = 1,\ t \ge 0$ (7)

(d) Determine the solution of the initial boundary-value problem

$u_t = 4u_{xx},\quad 0 < x < 1,\quad t > 0$

$u(x,0) = x(1-x),\quad 0 \le x \le 1$,

$u(0,t) = u(1,t) = 0,\ t > 0$ (7)