[This question paper contains 7 printed pages.]

2145 *Your Roll No.* ....

B.Sc. (Hons.) / III **C**

MATHEMATICS – Paper XII (V)

(Optimization)

(Admissions of 2009 and onwards)

Time : 3 Hours *Maximum Marks : 70*

(Write your Roll No. on the top immediately on receipt of this question paper.)

All *questions are compulsory.*
Choice is given within the questions.

1. Attempt any **two** parts :

(a) Solve the following problem by the method of constrained variation

Maximize $x_1 + x_2$

subject to $x_1^2 + x_2^2 = 1$. (5)

(b) Solve the following problem by the method of Lagrange multipliers and determine the character of the stationary point

Optimize $y = 2x_1^2 + 2x_1x_2 + x_2^2 - 20x_1 - 14x_2$

subject to $x_1 + 3x_2 \le 5$

$2x_1 - x_2 \le 4$ (5)

(c) Find the minimum along the direction of steepest descent of the function

$$y = x_1^2 + x_2^2$$

starting at the point (1, 1). (5)

2. Attempt any **two** parts :

(a) Determine the optimal functions that determine the stationary points for the following integral

$$I[y_1, y_2] = \int_{x_0}^{x_1} \left\{ \left[1 - (y_1')^2\right]^{-1/2} + 16y_2^2 - (y_2'')^2 \right\} dx \quad (5)$$

(b) Find the optimum functions that minimizes the integral

$$I[y_1(x), y_2(x)] = \int_{x_0}^{x_1} \left(y_1^2 + y_2^2\right) dx$$

subject to the differential equation constraint

$$\frac{dy_1}{dx} = y_2 - y_1 .$$ (5)

(c) In a production scheduling problem, the production rate is to be changed from 100 units per unit time to 300 units per unit time in ten times unit i.e. $p(0) = 100$ and $p(10) = 300$. For this problem the cost as a function of time is

$$C(t) = 2(p')^2 + 4tp'$$

where $p' = \frac{dp}{dt}$. Determine the production rate as a function of time that minimizes the cost over the time period. (5)

3. Attempt any two parts :

(a) Suppose $X^* = \begin{pmatrix} B^{-1}b \\ 0 \end{pmatrix}$ is a basic feasible solution of the linear programming problem minimize $z = cX$ subject to $AX = b$, $X \geq 0$. Prove that X^* is an optimal basic feasible solution if $z_j - c_j \leq 0$ for all non-basic variables. (8)

(b) Use Big-M method to show the following linear programming problem is unbounded

Minimize $z = -x_1 - x_2$

subject to $x_1 - x_2 - x_3 = 1$

$$-x_1 + x_2 + 2x_3 - x_4 = 1$$

$$x_1, x_2, x_3, x_4 \geq 0.$$ (8)

(c) Use simplex method to solve the system of equations

$$2x_1 + x_2 = 1$$

$$3x_1 - 2x_2 = 2$$

and hence write the inverse of the coefficient matrix $\begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix}$. (8)

4. Attempt any **two** parts :

(a) Obtain the dual of the following linear programming problem

Maximize $z = 8x_1 + 3x_2 - 2x_3$

subject to $x_1 - 6x_2 + x_3 \geq 2$

$$5x_1 + 7x_2 - 2x_3 = -4$$

$x_1 \leq 0,\ x_2 \geq 0,\ x_3$ unrestricted

with one of the dual variables being unrestricted in sign. (5)

(b) Find an optimal solution to the following problem by solving its dual

Minimize $z = -3x_1 - 6x_2$

subject to $4x_1 + 2x_2 \le 4$

$x_1 - x_2 \ge -\frac{2}{3}$

$x_1 \ge 0,\ x_2 \ge 0$ (5)

(c) Prove that if the primal problem has a finite optimal solution then the dual also has a finite optimal solution and the two optimal objective values are equal. (5)

5. Attempt any **three** parts :

(a) Solve the cost minimizing transportation problem :

	D_1	D_2	D_3	D_4	a_i
O_1	19	30	50	10	7
O_2	70	30	40	60	9
O_3	40	8	70	20	18
b_j	5	8	7	14	

(8)

(b) Solve the following cost minimizing assignment problem :

	I	II	III	IV	V	VI
A	5	4	7	9	1	2
B	3	8	5	4	9	6
C	2	8	1	9	7	5
D	8	6	5	7	9	3
E	7	4	9	8	3	5
F	5	6	8	7	4	9

(8)

(c) (i) Use the minimax criterion to find the best strategy for each player

$$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 4 & 0 \\ 3 & -2 & -1 \end{bmatrix}$$

(4)

(ii) Use relation of dominance to solve the game whose pay-off matrix is

$$\begin{bmatrix} 1 & 7 & 2 \\ 6 & 2 & 7 \\ 5 & 1 & 6 \end{bmatrix}$$

(4)

(d) (i) Solve graphically, the rectangular game whose pay-off matrix is

$$\begin{bmatrix} 2 & 1 & 0 & -2 \\ 1 & 0 & 3 & 2 \end{bmatrix} \qquad (4)$$

(ii) Reduce the following game into its corresponding primal and dual linear programming problems :

$$\begin{bmatrix} -2 & 3 & -1 \\ 4 & -1 & 2 \\ 1 & 2 & 3 \end{bmatrix} \qquad (4)$$