This question paper contains 8+3 printed pages]

Your Roll No................

2142

B.Sc.(Hons.)/III C

MATHEMATICS—Paper XII (*ii*)

(Discrete Mathematics)

(Admissions of 2009 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

All six questions are compulsory.

Do any *two* parts from each question.

1. (*a*) Let P and Q be ordered sets. Prove that $(a_1, b_1) \prec (a_2, b_2)$ in $P \times Q$ if and only if ($a_1 = a_2$ and $b_1 \prec b_2$) or ($b_1 = b_2$ and $a_1 \prec a_2$). 6

(*b*) (*i*) Let P be an ordered set and $x, y \in P$. Prove that the following are equivalent :

I. $x \leq y$;

P.T.O.

II. $\downarrow x \subseteq \downarrow y$;

III. $(\forall\ Q \in O(P))\ y \in Q \Rightarrow x \in Q$.

(*ii*) Draw the diagram of $M_3 \times 2$, where 2 denotes chain of two elements. 4, 2

(*c*) (*i*) Let f be a monomorphism from the lattice L into the lattice M. Show that L is isomorphic to a sublattice of M.

(*ii*) Let P be an ordered set and S, T $\subseteq$ P. Assume that $\vee S$, $\vee T$, $\wedge S$ and $\wedge T$ exist in P. Prove that if $S \subseteq T$, then $\vee S \leq \vee T$ and $\wedge S \geq \wedge T$. 4, 2

2. (*a*) Let B be a Boolean algebra. Prove that an ideal M in B is maximal if and only if for any $b \in B$, either $b \in M$ or $b' \in M$, but not both, hold. 6½

(*b*) (*i*) Find the conjunctive normal form of

$$(x_1 + x_2 + x_3)\ (x_1 x_2 + x_1' x_3)'.$$

(*ii*) Let L be a lattice. Prove that the following are equivalent :

$$(\mathrm{D})\ (\forall a, b, c \in \mathrm{L}), a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c);$$

$$(\mathrm{D})'\ (\forall p, q, r \in \mathrm{L}), p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r).$$

3½,3

(*c*) (*i*) Use Karnaugh diagram to simplify the following polynomial :

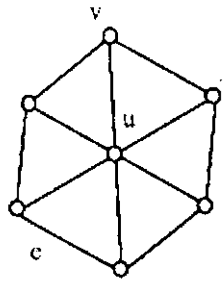
$$p = (x_1 + x_2)(x_1 + x_3) + x_1\ x_2\ x_3.$$

(*ii*) Determine the symbolic representation of the circuit given by :

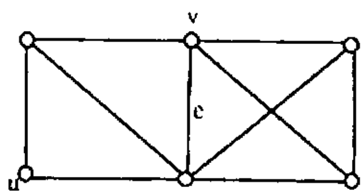
$p = (x_1 + x_3)'\ (x_1' + (x_2 + x_3)(x_2' + x_3'))$ using six gates. 3½,3

3. (*a*) (*i*) Define subgraph of a given graph. For each of the graphs below, draw pictures of the subgraphs $G \setminus \{e\}$, $G \setminus \{v\}$, $G \setminus \{u\}$

(*a*)

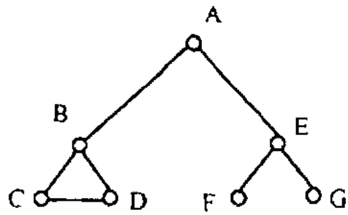


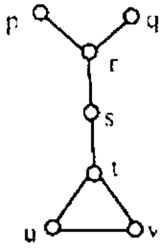
(*b*)



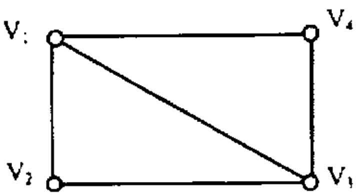
(*ii*) Show that the following graphs are isomorphic :

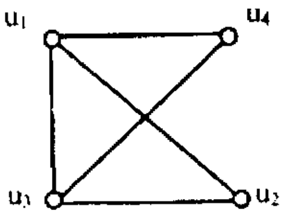
(*a*)



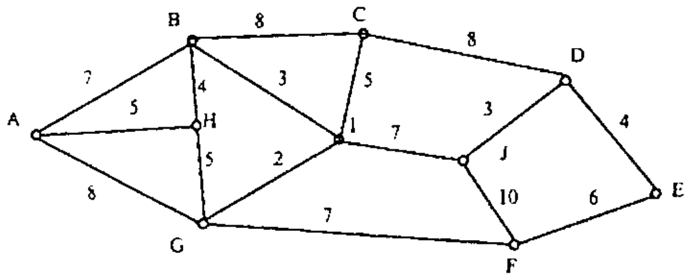
(*b*)  3,3

(*b*) Find the adjacency matrices A_1 and A_2 of the graphs G_1 and G_2 given below. Find a permutation matrix 'P' such that $A_2 = PA_1P^T$. 6

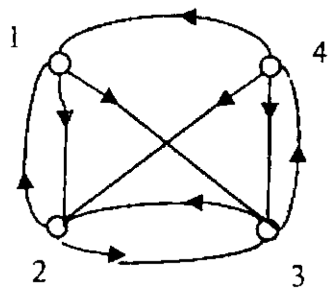




(c) Apply first form of Dijkstra's algorithm to find the shortest path from A to E in the following graph. Write steps. 6



4. (a) Let A be the adjacency matrix of the digraph : 6



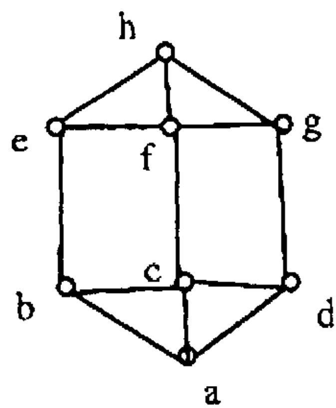
(i) Find A;

(*ii*) Is the digraph strongly connected ? Explain.

(*iii*) Is the digraph Eulerian ? Explain.

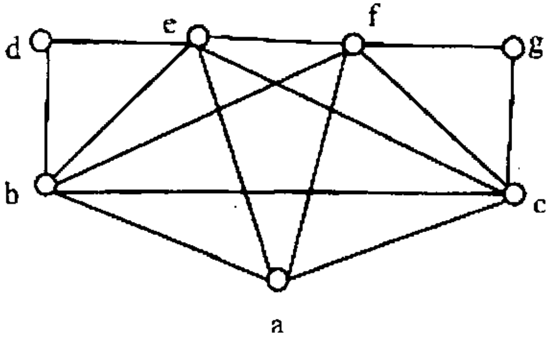
(*b*) Prove that a tree with more than one vertex has at least two leaves. Hence prove that any edge added to a tree must produce a cycle. 6

(*c*) Solve the Chinese Postman problem for the following graph : 6

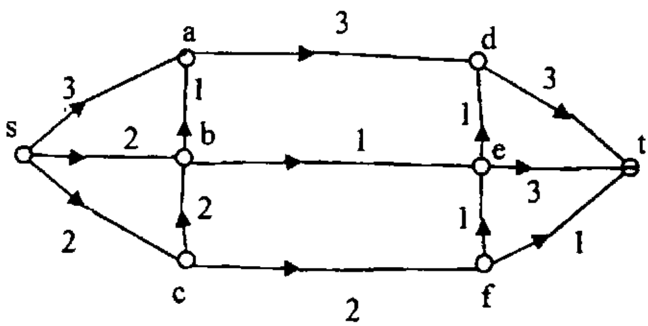


5. (*a*) Let G be a connected planar graph with $V \geq 3$ vertices and E edges. Then prove that $E \leq 3V - 6$. Hence show that K_5 is not planar. 6½

(*b*) (*i*) Compute $\chi(G)$ for the graph shown. Explain your answer and exhibit a $\chi(G)$ coloring.



(*ii*) Is $\chi(K_{m,n})$ the minimum of *m* and *n* ? Give reasons. 4,2½ (*c*) Find the maximum flow for the network shown and verify the answer by finding a cut whose capacity equals the value of the flow. 6½



6. (*a*) (*i*) Define $S^{-1}a$, ∇a and Δa . For the numeric function '*a*' given by

$$a_r = \begin{cases} 0, & 0 \le r \le 2 \\ 2^{-r} + 5, & r \ge 3 \end{cases}$$

show that $S^{-1}(\nabla a) = \Delta a$

(*ii*) Let '*a*' and '*b*' be the numeric functions such that :

$$a_r = 2^r,\ r \geq 0$$

and
$$b_r = \begin{cases} 0, & 0 \leq r \leq 2 \\ 2^r, & r \geq 3 \end{cases}$$

Find the convolution $a * b$. 3½, 3

(*b*) (*i*) Find a simple expression for the generating function of the discrete numeric function 0×1, 1×2, 2×3, 3×4...

(*ii*) Determine the discrete numeric function corresponding to the generating function :

$$A(z) = \frac{1}{5 - 6z + z^2}.$$ 3½,3

(*c*) (*i*) The solution of the recurrence relation

$a_r = Aa_{r-1} + B3^{r-1}, r \geq 1$ is $a_r = C2^r + D3^{r-1}, r \geq 0$.

Given that $a_0 = 19$ and $a_1 = 50$, determine the constants A, B, C and D

(*ii*) Solve the recurrence relation $a_r + 6a_{r-1} + 9a_{r-2} = 3$, given that $a_0 = 0$ and $a_1 = 1$.

3½,3