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Your Roll No.....

6101

**B.Sc. (Hons.)/Sem. I** **B**

**MATHEMATICS—Paper I.1**

(Calculus)

(Admissions of 2011 and onwards)

Time : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the Sections are compulsory.

Use of scientific calculator is allowed.

**Section I**

Attempt any *Four* questions from Section I.

1. Find the  $n$ th derivative of  $e^x \sin^4 x$ . 5
2. Sketch the graph of the curve in polar coordinates. 5

$$r = 2 \cos 3\theta.$$

3. Find the vertical and horizontal asymptotes of the curve

$$y = \frac{1}{x-1}, \quad (x \neq 1) \quad 5$$

4. Find

$$\lim_{x \rightarrow \infty} x^5 \left( \sin \frac{1}{x} - \frac{1}{x} + \frac{1}{6x^3} \right) \quad 5$$

P.T.O.

5. A manufacturer estimates that when  $x$  units of a particular commodity are produced each month, the total cost (in dollars) will be

$$C(x) = \frac{1}{8}x^2 + 4x + 200$$

and all units can be sold at a price of  $P(x) = 49 - x$  dollars per unit. Determine the price that corresponds to the maximum profit. 5

### Section II

Attempt any *Four* questions from Section II.

6. Evaluate :

$$\int_0^{\pi/3} \sin^4 3x \cos^3 3x \, dx. \quad 5$$

7. Find the volume of the solid that results when the region enclosed by the curve is revolved about the  $x$ -axis

$$y = x^2, \quad x = 0, \quad x = 2, \quad y = 0. \quad 5$$

8. Find the volume of the solid that results when the region enclosed by  $x = y^2$  and  $x = y$  is revolved about the line  $y = -1$ . 5

9. Use cylindrical shells to find the volume of the solid generated when the region R under  $y = x^2$  over the interval  $[0, 2]$  is revolved about the  $x$ -axis. 5
10. Find the area of the surface generated by revolving the parametric curve  $x = t^2, y = 2t, 0 \leq t \leq 4$  about the  $x$ -axis. 5

### Section III

Attempt any *Three* questions from Section III.

11. Identify and sketch the curve  $xy = 1$ . 5
12. Let an  $x'y'$ -coordinate system be obtained by rotating an  $xy$ -coordinate system through an angle of  $\theta = 60^\circ$ . 5
- (a) Find the  $x'y'$ -coordinates of the point whose  $xy$ -coordinates are  $(-2, 6)$ .
- (b) Find an equation of the curve  $\sqrt{3}xy + y^2 = 6$  in  $x'y'$ -coordinates. 5
13. Rotate the coordinate axes to remove the  $xy$ -term. Then name the conic and sketch its graph :  

$$9x^2 - 24xy + 16y^2 - 80x - 60y + 100 = 0.$$
 5
14. Derive the equation of the conic in polar form

$$r = \frac{ed}{1 + r \cos \theta}$$

5

## Section IV

Attempt any *Four* questions from Section IV.

15. If  $\vec{v}$  and  $\vec{w}$  are non-zero vectors in  $\mathbb{R}^3$  with  $\theta$  the angle between  $\vec{v}$  and  $\vec{w}$  ( $0 \leq \theta \leq \pi$ ), then show that :

$$\|\vec{v} \times \vec{w}\| = \|\vec{v}\| \|\vec{w}\| \sin \theta. \quad 5$$

16. Find the tangent vector to the graph of the given vector function  $\vec{F}$  at the points :

$$\vec{F}(t) = t^2 \hat{i} + 2t \hat{j} + (t^3 + t^2) \hat{k};$$

$$t = 0, t = 1, t = -1. \quad 5$$

17. State and prove Kepler's Second Law. 5
18. A boy standing at the edge of a cliff throws a ball upward at an angle of  $30^\circ$  with the horizontal axis and an initial speed of 64 ft/s. Suppose that when the ball leaves the boy's hand, it is 48 ft. above the ground at the base of the cliff. What are the time of flight of the ball and its range ? 5
19. The position vector of a moving body is  $\vec{R}(t) = 2t \hat{i} - t^2 \hat{j}$  for  $t \geq 0$ . Express  $\vec{R}$  and the velocity vector  $\vec{V}(t)$  in terms of  $u_r$  and  $u_\theta$ . 5