This question paper contains 7 printed pages]

Roll No. ☐☐☐☐☐☐☐☐☐☐

S. No. of Question Paper : 8311

Unique Paper Code : 235161   C

Name of the Paper : MACT-303 : Mathematics and Statistics

Name of the Course : B.Sc. (H) Bot./Zool and B.Sc. Life Science

Semester : III/I

Duration : 3 Hours   Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

There are three Sections in this question paper.

Attempt any *two* questions from each Section.

Students are allowed to use simple calculators.

Section I

1.   (a)   Draw the graph of $y = 1 - 2x - x^2$. Also find the maximum value of y.   6

     (b)   Discuss the continuity of the given function at $x = 0$.

$$f(x) = \begin{cases} x - 1 & x < 0 \\ 2 & x = 0 \\ x^2 & x > 0 \end{cases}$$

                                                   5½

     (c)   If $y = ae^{2x} + bxe^{2x}$, prove that :

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0.$$

                                                   3

(a) A particle moves on a line away from its initial position, so that after t hours it is $s = 3t^2 + t$ km from its initial position. Find the average velocity of the particle over the interval [1, 3]. 5½

(b) Write down the first five terms of the sequence given by the recursion formula :

$$a_1 = 1, \qquad a_n = na_{n-1}.$$ 5½

(c) 6000 persons were tested for A,B,O, blood groups, 2527 had the antigen A, 2234 the antigen B and 1846 no antigen. How many individuals had both antigen ? 5½

(a) Find $\dfrac{d^2y}{dx^2}$

if :

(i) $y = x^5 + 4\sin x + 2e^x$

(ii) $y = \log(x^2 + 1)$. 6

(b) Examine the nature of the roots of the following equation :

$$x^2 - 8x + 16 = 0.$$

Also find its roots. 4½

(c) Integrate the following :

(i) $\int x \sin x \, dx$

(ii) $\int \dfrac{x}{1+x^2} \, dx$

(iii) $\int \left(x^3 + \dfrac{1}{x^2} \right) dx$. 6

4. (a) Evaluate :

$$\int_0^1 \frac{3x^3 - 4x^2 + 1}{\sqrt{x}} \, dx.$$

5½

(b) Find the sum of infinite geometric series :

$$1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots.$$

5

(c) The population of bacteria develops with a constant relative growth rate of 0.7944 per member per day. Initial population consists of 4 members. Find the population size after 6 days.

6

Section II

5. (a) If

$$A = \begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} a & b \\ 3 & 5 \end{pmatrix},$$

find the value of a and b such that AB = BA.

6

(b) If

$$A = \begin{bmatrix} 2 & 0 & 1 \\ -2 & 4 & 2 \\ -1 & 3 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 & 5 & 8 \\ 0 & -1 & 1 \\ 2 & 2 & 6 \end{bmatrix},$$

evaluate 2A − 3B.

5

(a) Find the value of x such that :

$$(1 \quad x \quad 1) \begin{pmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 2 & 2 & 6 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ x \end{pmatrix} = 0$$

6

(b) If

$$A = \begin{pmatrix} 1 & 5 \\ 3 & 2 \end{pmatrix} \text{ and } B = \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix},$$

verify $(A + B)^T = A^T + B^T$.

5

(a) What matrix has the effect of rotating every vector through 90° and then projecting the result on x-axis ?

5

(b) If

$$A = \begin{pmatrix} 2 & 0 & 1 \\ -2 & 4 & 2 \\ -1 & 3 & 4 \end{pmatrix} \text{ and } B = \begin{pmatrix} 4 & 5 & 8 \\ 0 & -1 & 1 \\ 2 & 2 & 6 \end{pmatrix},$$

find matrix X such that :

$$2A + 4B - 3X = 0.$$

6

Section III

(a) Find the missing frequencies for the distribution of marks of 100 pupils, median marks is known to be 32.　　5

Marks	No. of pupils
0–10	10
10–20	—
20–30	25
30–40	30
40–50	—
50–60	10

(b) The following frequency table gives the ages (in years) of a group of 50 children invited to a birthday party. Find the standard deviation of the distribution and coefficient of variation :　　5

Age (in years)	Frequency
5–7	16
7–9	13
9–11	10
11–13	6
13–15	5

1. (a) Fit a straight line to the following data taking y as the dependent variable :   5

X	Y
1	3
2	5
3	12
4	16
5	24

(b) In a partially destroyed laboratory record of an analysis of a correlation data, only the following two regression lines are available :   5

$$8x - 10y + 66 = 0 \text{ and } 40x - 18y = 214.$$

Find :

(i) the average values of x and y

(ii) the correlation coefficient

(iii) variance of y if var(x) = 9

(iv) the covariance of x and y.

(a) For the probability distribution given below, find k and E(X) : 5

x	$p(x)$
1	k
2	$2k$
3	$3k$
4	$4k$
5	$5k$
6	$6k$

(b) The means of two samples of 1000 and 2000 members are 67.5 inches and 68.0 inches respectively. Can the samples be regarded as drawn from the population of standard deviation 2.5 inches ? 5

(a) Find the binomial probability distribution whose mean is 3 and variance 2. 5

(b) In a certain factory manufacturing razor blades, there is a small chance 1/500 for any blade to be defective. The blades are in the packets of 10. Use Poisson distribution to find the probability that the packet contains one defective blade. 5