[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 8839 **C** **Roll No............**

Unique Paper Code : 237161

Name of the Paper : STC-301 : Basic Statistics and Probability

Name of the Course : B.Sc. (Mathematical Sciences), Part I (Sem. I)
B.Sc. (Hons.) Computer Science, Part II (Sem. III)

Semester : I / III

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** Sections.

SECTION I

Attempt **all** questions. **All** questions carry equal marks.

1. The mean and variance of a normal distribution are 60 and 25 respectively. Find the inter-quartile range and the mean deviation of the distribution.

2. For a group of 20 items, $\sum X = 1452$, $\sum X^2 = 144280$ and mode is 63.7. Find the Pearsonian coefficient of skewness.

3. Give an example to show that pair wise independent events may not be jointly independent.

4. Suppose that an airplane engine will fail, when in flight, with probability 1-p independently from engine to engine. Suppose that the airplane will make a successful flight if atleast 50 percent of its engines remain operative. For what values of p is a four engine plane preferable to a two engine plane ?

5. Let X be a continuous random variable with probability density function f(x) :

$$f(x) = \begin{cases} c(1-x^2), & -1 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

where c is a constant. Find

(i) the value of c

(ii) the cumulative distribution function F(X) of x.

(iii) $P(1/2 < X < 3/2) = ?$

6. For any random variables X, Y, Z and constant k, prove the following :

(a) $Cov(X,X) = Var(X)$

(b) $Cov(kX,Y) = k\ Cov(X,Y)$

(c) $Cov(X,Y+Z) = Cov(X,Y) + Cov(X,Z)$

7. If the probability is 0.40 that a child exposed to a certain contagious disease will catch it, what the probability that the tenth child exposed to the disease will be the third to catch it.

SECTION II

Attempt any **two** questions. **All** questions carry equal marks.

1. The number of workers employed, the mean wage in rupees per month and standard deviation in each section of a company are given below. Calculate the mean wages and standard deviation for all workers taken together.

Section	No. of workers employed	Mean wage	Standard deviation
A	50	113	6
B	60	120	7
C	90	115	8

2. The median and mode of the following wage distribution are known to be rupees 33.5 and rupees 34 respectively. Three frequency values from the table are missing. Find these missing values (X,Y, Z) given that N (total frequency) is equal to 230.

Daily wages (in rupees')	Frequencies
0–10	10
10–20	10
20–30	X
30–40	Y
40–50	Z
50–60	6
60–70	4

3. If the first three moments of distribution about the value 5 are equal to –4, 22 and –117, determine the corresponding moments :

(i) about mean

(ii) about zero

SECTION III

Attempt any **six** questions. **All** questions carry equal marks.

1. A laboratory blood test is 95% effective in detecting a certain disease when it is, infact, present. However the test also yields a 'false positive' result for 1% of the healthy persons tested. If 0.5% of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive ?

2. Suppose that all n men at a party throw their hats in the centre of the room. Each man then randomly selects a hat. Show that the probability that none of the men selects his own hat is

$$\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\ldots\ldots+\frac{(-1)^n}{n!}.$$

3. If X and Y are independent Gamma variables with parameters α and β respectively, show that $U = X + Y$ and $Z = X/(X + Y)$ are independent and identify the distributions of U and Z.

4. Let $X_1, X_2, \ldots\ldots X_n$ be independent random variables, each having a uniform distribution over (0, 1). Let M = maximum $(X_1, X_2, \ldots\ldots X_n)$. Show that the distribution function of M, $F_M(.)$, is given by

$$F_M(x) = x^n, \qquad 0 \leq x \leq 1$$

What is the probability density function of M ?

5. Show that Poisson distribution can be derived as a limiting case of Binomial distribution, stating clearly the approximate conditions.

6. Define moment generating function of a discrete and a continuous random variable. Find the moment generating function of an exponential distribution (parameter λ) and hence find the mean and variance of an exponential distribution.

7. Find the mode of the binomial distribution.