This question paper contains 4 printed pages]

Roll No. [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

S. No. of Question Paper : **8801**

Unique Paper Code : **235101** **C**

Name of the Paper : **I.1 : Calculus–I**

Name of the Course : **B.Sc. (Hons.) Mathematics (Part I)**

Semester : **I**

Duration : **3 Hours** Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the sections are compulsory.

Use of non-programmable scientific calculator is allowed.

All questions carry equal marks.

Section I

Attempt any *four* questions from Section I.

1. If $y = \sin(m \sin^{-1} x)$, show that $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 - m^2)y_n = 0$.

2. Find horizontal asymptote of the curve $f(x) = \left(\dfrac{x+5}{x+4}\right)^{3x}$.

3. Find constant a and b so that $\lim\limits_{x \to 0}\left(\dfrac{\sin 2x}{x^3} + \dfrac{a}{x^2} + b\right) = 1$.

P.T.O.

4. If the total cost of manufacturing x units of a commodity is $C(x) = 3x^2 + 5x + 75$, then :

(a) At what level of production is the average cost per unit the smallest ?

(b) At what level of production is the average cost per unit equals the marginal cost ?

5. Sketch the curve $r^2 = 16\sin 2\theta$ in polar coordinates.

Section II

Attempt any *four* questions from Section II.

6. Show that

$$\int_0^{\pi/3} \cos^4 3\theta \sin^2 6\theta \, d\theta = \frac{5}{96}\pi.$$

7. The base of a certain solid is the region enclosed by $y = \sqrt{x}$, $y = 0$ and $x = 4$. Every cross-section perpendicular to the x-axis is a semi-circle with its diameter across the base. Find the volume of the solid.

8. Find the volume of the solid generated when the region enclosed by $y = \sqrt{x+1}$, $y = \sqrt{2x}$ and $y = 0$ is revolved about x-axis.

9. Find the arc length of the parametric curve $x = \cos t + t\sin t$, $y = \sin t - t\cos t$ for $0 \le t \le \pi$.

10. Find the area of the surface generated by revolving the curve $x=\sqrt{9-y^2}, -2 \le y \le 2$ about y - axis.

Section III

Attempt any *three* questions from Section III.

11. Find the equation of the hyperbola which passes through the origin and has asymptotes $y=2x+1$ and $y=-2x+3$.

12. Describe the graph of the equation $4x^2+8y^2+16x+16y+20=0$.

13. Trace the conic $x^2-3xy+y^2+10x-10y+21=0$ by rotating the coordinate axes to remove the *xy*-term.

14. Derive the equation of conic in polar form by taking directrix parallel to the polar axis and is on either side of the pole.

Section IV

Attempt any *four* questions from Section IV.

15. Find $\lim_{t \to 0^+} \vec{F}(t)$, where $\vec{F}(t) = \left[\frac{\sin 3t}{\sin 2t}\hat{i} + \frac{\log(\sin t)}{\log(\tan t)}\hat{j} + t \log t\,\hat{k}\right]$.

16. Sandy hits a baseball at a 30° angle with a speed at 144 ft/s. If the ball is 4 ft above the ground level when it is hit, what is the maximum height reached by the ball ? How far will it travel from home plate before it lands ? If it just barely clears a 5 ft wall in the outfield before landing, how far is the wall from home plate ?

17. Find the position vector $\vec{R}(t)$ and velocity vector $\vec{V}(t)$, where the acceleration vector is $\vec{A}(t) = (\cos t)\hat{i} - (t \sin t)\hat{k}$ and initial position and velocity vectors are $\vec{R}(0) = \hat{i} - 2\hat{j} + \hat{k}$; $\vec{V}(0) = 2\hat{i} + 3\hat{k}$.

18. The position vector of a moving object is $\vec{R}(t) = \sin t\,\hat{i} + \cos t\,\hat{j} + \sin t\,\hat{k}$. Find the tangential and normal component of the acceleration of the object at time t.

19. An object moves along the curve $r = \sin\theta$, $\theta = 2t$. Find its velocity and acceleration in terms of unit polar vectors u_r and u_θ.