[This question paper contains 2 printed pages.]

Sr. No. of Question Paper : 6210 **D** **Your Roll No...............**

Unique Paper Code : 235362

Name of the Course : **B.Sc. (H) Physics**

Name of the Paper : Mathematics–I / [PHHT 310]

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.

1. (a) Define a Cauchy sequence. Prove that every Cauchy sequence is bounded. Is the converse true. Justify your answer by giving an example. (7½)

 (b) State necessary and sufficient condition for the convergence of monotonic sequence. Use this to prove that

$$a_n = \frac{1}{n+1} + \frac{1}{n+2} \ldots\ldots\ldots + \frac{1}{n+n}$$

 is convergent. (7½)

 (c) (i) Define limit superior and limit inferior of the sequence $\langle a_n \rangle$. Find the limit superior and the limit inferior of the following sequences

$$\langle 2,4,2,4,2,4 \ldots\ldots\ldots \rangle \text{ and } \left\langle (-1)^n \left(1 + \frac{1}{n}\right) \right\rangle$$ (2,2)

 (ii) Prove that $a_n \to a$ implies that $|a_n| \to |a|$. (3½)

2. (a) Check for convergence the following series

 (i) $1 + \frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \ldots\ldots\ldots, x > 0$

 (ii) $\frac{1}{1.2.3} - \frac{1}{2.3.4} + \frac{1}{3.4.5} - \ldots\ldots\ldots$ (4½,3)

 (b) Show that the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$ is convergent if $p > 1$ and divergent if $0 < p \le 1$. (7½)

P.T.O.

(c) Discuss continuity of the function f(x) defined by

$$f(x) = \begin{cases} 1, & x \text{ is rational} \\ -1, & x \text{ is irrational} \end{cases}$$ (7½)

3. (a) If a function f is continuous on a closed bounded interval [a,b], then show that it attains its bounds in [a,b]. (7½)

(b) Obtain Maclaurin's series expansion for f(x) = sin x. (7½)

(c) Show that f(x) = 1/x is not uniformly continuous in $(0,\infty)$.

4. (a) (i) State Young's Theorem. (1½)

(ii) Let $f(x,y) = \begin{bmatrix} \dfrac{x^3y}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{bmatrix}$

Prove or disprove $f_{xy}(0,0) = f_{yx}(0,0)$. (6)

(b) Find the expansion of the polynomial $x^3 + 2x^2y + y^3$ about (1,1) upto the terms of second degree. (7½)

(c) Investigate for maxima and minima the following function

$f(x,y) = x^3 + y^3 - 63(x+y) + 12xy$. (7½)

5. (a) Let f be a bounded function on [a,b]. If P and Q. are two partitions of [a,b] where P is contained in Q, then show that

$L(P, f) \leq L(Q, f) \leq U(Q, f) \leq U(P, f)$. (7½)

(b) (i) Show that the function f defined by

f(x) = 1, when x is rational,

= –1, when x is irrational,

is not Riemann integrable on any interval. (4)

(ii) Give an example of a function f defined on [0,1] which is not integrable but If 1 is integrable. (3½)

(c) Show that f(x) = 2x + 3 is integrable in [1, 2] and also find the value of the integral. (7½)

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