[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 6601 **D** **Your Roll No...............**

Unique Paper Code : 235101

Name of the Course : **B.Sc. (H) Mathematics – I**

Name of the Paper : Calculus – I / Paper Code : MAHT 101

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All the sections are compulsory.
3. Use of non-programmable scientific calculator is allowed.
4. **All** question carry equal marks.

SECTION – I

Attempt any **four** *questions from Section-I.*

1. If $y = (\sin^{-1} x)^2$, Prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$.

2. Find constants a and b that guarantee that the graph of the function defined by $f(x) = \dfrac{ax + 5}{3 - bx}$ will have a vertical asymptote at $x = 5$ and a horizontal asymptote at $y = -3$.

3. Find all values of A and B so that $\lim\limits_{x\to 0}\left(\dfrac{\sin Ax + Bx}{x^3}\right) = 36$.

4. It is projected that t years from now the population of a certain country will be $P(t) = 50e^{0.02t}$ million.

(a) At what rate will the population be changing with respect to time 10 years from now ?

(b) At what percentage rate will the population be changing with respect to time t years from now ?

5. Sketch the curve $r = 3\sin 2\theta$ in polar coordinates.

SECTION – II

Attempt any **four** *questions from Section-II.*

6. Evaluate $\int_0^{\pi/3} \sin^4 3\theta \cos^3 3\theta \, d\theta$.

7. Find the volume of the solid that results when the region enclosed by $x = y^2$ and $x = y$ is revolved about the line $y = -1$.

8. Find the volume of the solid generated when the region enclosed by $y = 9 - x^2$, $y = 0$ is revolved about the x-axis.

9. Find the arc length of the parametric curve $x = \cos 2t$, $y = \sin 2t$ for $0 \le t \le \pi/2$.

10. Find the area of the surface generated by revolving the curve $x = y^3$, $0 \le y \le 1$ about y-axis.

SECTION – III

Attempt any **three** *questions from Section-III.*

11. Find an equation for a hyperbola that satisfies the condition that the curve has vertices $(\pm 2,0)$, and foci $(\pm 3,0)$.

12. Describe the graph of the equation $x^2 + 9y^2 + 2x - 18y + 1 = 0$.

13. Trace the conic $xy + 9 = 0$ by rotating the coordinate axes to remove the xy term.

14. Find a polar equation for the ellipse that has its focus at the pole and satisfies the condition that 'directrix to the right of the pole; $a = 8$; $e = 1/2$'.

SECTION – IV

Attempt any **four** *questions from Section-IV.*

15. Find $\lim_{t\to 0} \vec{F}(t)$, where $\vec{F}(t) = \left[\frac{\sin t}{t}\hat{i} + \frac{1-\cos t}{t}\hat{j} + e^{1-t}\hat{k}\right]$.

16. A shell is fired at ground level with a muzzle speed of 280 ft/s and at an elevation of 45° from the ground level.

    (a) Find the maximum height attained by the shell.

    (b) Find the time of flight and the horizontal range of shell.

    (c) Find the velocity and speed of the shell at impact.

17. Find the position vector $\vec{R}(t)$ and velocity vector $\vec{V}(t)$, where the acceleration vector $A(t) = t^2\hat{i} - 2\sqrt{t}\hat{j} + e^{3t}\hat{k}$, initial position $R(0) = 2\hat{i} + \hat{j} - \hat{k}$ and initial velocity $V(0) = \hat{i} - \hat{j} - 2\hat{k}$ are given.

18. The velocity $V_0 = 2\hat{i} + 3\hat{j} - \hat{k}$ and the acceleration $A_0 = -\hat{i} - 5\hat{j} + 2\hat{k}$ of a moving object are given. Find the normal and tangential component of acceleration of the object at that instant.

P.T.O.

19. An object moves along the curve $r = e^{-\theta}$, $\theta = 1 - t$ in the polar plane. Find its velocity and acceleration in terms of the unit polar vectors u_r and u_θ.