

[This question paper contains 4 printed pages.]

**Sr. No. of Question Paper : 5002                                      D                                      Your Roll No.....**

Unique Paper Code                    : 237161

Name of the Course                 : **B.Sc. (Mathematical Sciences), Part I (Sem. I)**  
**B.Sc. Hons. (Computer Sciences), Part II (Sem. III)**

Name of the Paper                    : STC-301 : Basic Statistics and Probability

Semester                                : I / III

Duration : 3 Hours

Maximum Marks : 75

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** sections.
3. **All** questions carry equal marks.

**SECTION I**

*(Attempt all questions. All questions carry equal marks.)*

1. Let random variable  $X$  be normally distributed with mean  $\mu$  and variance  $\sigma^2$ . Obtain the moment generating function of  $X$ .
2. In answering a question on a multiple choice test a student either knows the answer or he guesses. Let  $p$  be the probability that he knows the answer and  $1-p$  be the probability that he guesses the answer. Assume that a student who guesses an answer will be correct with probability  $1/5$ , where 5 is the number of multiple choice alternatives. What is the conditional probability that a student knew the answer to a question given that he answered it correctly ?
3. Let  $X$  be  $U(0,1)$ . Calculate  $E[X^3]$ .

*P.T.O.*

4. Suppose that the moment generating function of a random variable  $X$  is given by  $M_X(t) = e^{3(e^t-1)}$ . What is the  $P\{X = 0\}$  ?

5. Let  $X$  be a random variable with probability density

$$f(x) = \begin{cases} ce^{-2x}, & 0 < x < \infty \\ 0, & x < 0 \end{cases}.$$

(i) Find the value of  $c$  ?

(ii) What is  $P\{X > 2\}$  ?

6. The coefficients of variation of wages of male and female workers are 55% and 70% respectively, while the standard deviations are 22.0 and 15.4 respectively. Calculate the overall average of all workers given that 80% of the workers are male.

7. In a frequency distribution, the coefficient of skewness based on quartiles is 0.5. If the sum of the upper and lower quartiles is 28 and the median is 11, find the values of lower and upper quartiles.

## SECTION II

*(Attempt any two questions. All questions carry equal marks.)*

1. The mean and standard deviation of scores of a group of 200 students were found to be 40 and 15 respectively. Later on it was discovered that the scores 43 and 35 were misread as 34 and 53 respectively. Find the corrected mean and standard deviation corresponding to the corrected figures.

2. The first four moments of distribution about  $x = 4$  are 1, 4, 10 and 45. Obtain the various characteristics of the distribution on the basis of the above information. Comment on the nature of the distribution.

3. Calculate the median for the following data, if the mean value is 45.

|           |      |       |       |       |       |       |       |       |
|-----------|------|-------|-------|-------|-------|-------|-------|-------|
| Marks     | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 |
| Frequency | 5    | 8     | 7     | 12    | f     | 20    | 10    | 10    |

### SECTION III

(Attempt any six questions. All questions carry equal marks.)

- An urn contains  $b$  black balls and  $r$  red balls. One of the balls is drawn at random, but when it is put back in the urn,  $c$  additional balls of the same colour are put with it. Suppose another ball is drawn, then prove that the probability that the first ball drawn was black given that second ball drawn was red is  $b/(b+r+c)$ .
- Suppose that each of three men at a party throws his hat into the center of the room. The hats are first mixed up and then each man randomly selects a hat. What is the probability that at least one man selects his own hat?
- Show that the expected number of independent trials needed to be performed until the first success is attained is equal to the reciprocal of the probability that any one trial results in a success.
- If the probability is 0.75 that an applicant for a driver's license will pass the road test on any given try, what is the probability that an applicant will finally pass the test on the fourth try?
- Suppose that  $X_1, X_2, \dots, X_n$  are independent and identically distributed random variables with mean  $\mu$  and variance  $\sigma^2$ . Then show that
  - $E(\bar{X}) = \mu$
  - $\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$
  - $\text{Cov}(\bar{X}, X_i - \bar{X}) = 0; i = 1, 2, \dots, n.$

P.T.O.

6. Calculate  $\text{Var}(X)$  when  $X$  represents the outcome when a fair die is rolled.
7. State and prove the additive property of Poisson distribution.