This question paper contains **4+2** printed pages]

Roll No. | | | | | | | | | | |

S. No. of Question Paper : **17**

Unique Paper Code : **236163/234506** **G**

Name of the Paper : **Operational Research—I (Concurrent)**

Name of the Course : **B.Sc. Mathematical Sciences & B.Sc. (H) Computer Science**

Semester : **I/V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *Five* questions in all.

Question No. **1** is compulsory.

Use of non-programmable calculator is allowed.

1. (*i*) Explain the nature of O.R. and its limitations.

(*ii*) What is shadow price ? Write the dual of the following LPP :

Minimize : $Z = x_1 + x_2 + x_3$

Subject to :

$$x_1 - 3x_2 + 4x_3 = 5$$

$$x_1 - 2x_2 + 7x_3 \leq 3$$

$$2x_2 - x_3 \geq 4$$

$x_1 \geq 0$ x_2 and x_3 unrestricted in sign

P.T.O.

(*iii*) Consider the following LP with two variables :

Maximize : $Z = 3x_1 + 2x_2$

Subject to :

$$x_1 + x_2 \leq 4$$

$$x_1 - x_2 \leq 2$$

$$x_1, x_2 \geq 0.$$

(*a*) Determine all the basic solutions of the problem, and classify them as feasible and infeasible.

(*b*) Verify optimal solution graphically. Show how infeasible basic solutions are represented on graphical solution space.

(*iv*) Consider the following LPP :

Maximize : $Z = 5x_1 - 6x_2 + 3x_3 - 5x_4 + 12x_5$

Subject to :

$$x_1 + 3x_2 + 5x_3 + 6x_4 + 3x_5 \leq 30$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0.$$

Solve the problem by inspection (do not use Gauss-Jordan row operations), and justify the answers in terms of the basic solutions of the simplex method.

(*v*) What do you understand by the convex set. Show that the following set is convex :

$$C = \{(x_1, x_2) \mid x_1^2 + x_2^2 \leq 3, x_1 \geq 0, x_2 \geq 0\}.$$

(*vi*) A manufacturing company purchases 9000 parts of a machine for its annual requirements, ordering one month usage at a time. Each part costs Rs. 20. The ordering cost per order is Rs. 15 and the carrying charges are 15% of the average inventory per year. You have been assigned to suggest a more economical purchasing policy for the company. What advice would you offer and how much would it save the company per year ?

(*vii*) Explain ABC analysis. What are its advantages and limitations, if any ? 7×5=35

2. In the Ma-and-Pa grossery store, shelf space is limited and must be used effectively to increase profit. Two cereal items grano and wheat compete for a total shelf space of 60 sq. feet. A box of grano occupies 0.2 sq. feet and a box of wheat needs 0.4 sq. feet. The maximum daily demand of grano and wheat are 200 and 120 boxes respectively. A box of grano nets $ 1 in profit and a box of wheat $ 1.35. Ma-and-Pa thinks that because the unit profit of wheat is 35% higher than that of grano, wheat should be allocated 35% more space than grano which amounts to allocating about 57% to wheat and 43% to grano. What do you think ? Formulate the above problem as linear programming problem and solve it graphically. 10

3. Solve the following LPP using two-phase method :

Maximize : $Z = 2x_1 + 2x_2 + 4x_3$

Subject to :

$$2x_1 + x_2 + x_3 \leq 2$$

$$3x_1 + 4x_2 + 2x_3 \geq 8$$

$$x_1, x_2, x_3 \geq 0$$

P.T.O.

(*a*) Show that phase I will terminate with an artificial variable in the basis.

(*b*) Remove artificial variable prior to the start of phase II, then carry out phase II. 10

4. (*a*) The following table represents a simplex iteration. All variables are non-negative. The table is not optimal for either a maximisation or minimization problem. Thus, when a non-basic variable enters the solution it can either increase or decrease z or leave it unchanged, depending on the parameters of the entering non-basic variable.

Basic	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	**Solution**
x_8	0	3	0	–2	–3	–1	5	1	12
x_3	0	1	1	3	1	0	3	0	6
x_1	1	–1	0	0	6	–4	0	0	0
z-row	0	–5	0	4	–1	–10	0	0	620

Categorize the variables as basic and non-basic and provide the current values of all the variables. Assuming that the problem is of the maximisation type, identify the non-basic variables that have the potential to improve the value of z. If each such variable enters the basic solution, determine the associated leaving variable, if any, and the associated change in z. Do not use the Gauss-Jordan row operation.

(*b*) Consider the following Linear Programming Problem :

Maximize : $Z = 5x_1 + 2x_2 + 3x_3$

Subject to :

$$x_1 + 5x_2 + 2x_3 = 30$$

$$x_1 - 5x_2 - 6x_3 \leq 40$$

$$x_1, x_2, x_3 \geq 0$$

Given that the artificial variable x_4 and the slack variable x_5 form the starting basic variables and that M equals 100 when solving the problem, the optimal table is given in the following table. Write the associated dual problem and determine its optimal solution. 2×5=10

Basic	x_1	x_2	x_3	x_4	x_5	Solution
x_1	1	5	2	1	0	30
x_5	0	−10	−8	−1	1	10
z-row	0	23	7	105	0	150

5. Solve the following LPP using simplex algorithm and hence list out five optimal solutions using alternative optimal solution : 10

Maximize : $Z = 2x_1 + 4x_2$

Subject to :

$$x_1 + 2x_2 \leq 5$$

$$x_1 + x_2 \leq 4$$

$$x_1, x_2 \geq 0.$$

6. (*a*) A company manufactures two products A and B. The unit revenues are \$ 2 and \$ 3, respectively. Two raw materials M1 and M2, used in the manufacture of the two products have daily availability of 8 and 18 units, respectively. One unit of A uses 2 units of M1 and 2 units of M2 and the respective values for B are 3 and 6.

(*i*) Determine the dual prices of M1 and M2 and their feasibility ranges graphically.

(*ii*) Is it advisable to arrange 2 additional units of M1 at the cost of 25 cents per unit ?

(*b*) Write a short note on degeneracy of LPP. 7,3

7. (*a*) Derive the expression for EOQ under the following assumptions :

(*i*) demand is known and uniform

(*ii*) shortages are not allowed

(*iii*) lead time is zero.

(*b*) Find the optimum order quantity for a product for which the price breaks are as follows :

Quantity	Unit Cost (in Rs.)
$0 \leq Q_1 < 800$	Re. 1.00
$800 \leq Q_2$	Re. 0.98

The yearly demand for the product is 1600 units per year, cost of placing an order is Rs. 5, the cost of storage is 10% per year. 2×5=10