

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1789

FC-3

Your Roll No.....

Unique Paper Code : 32351102

Name of the Paper : C2 – Algebra

Name of the Course : B.Sc. (H) Mathematics – I (CBCS)

Semester : I

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All six questions are compulsory.
3. Do any two parts from each question.

1. (a) Find the polar representation of the complex number (6)

$$z = 1 + \cos\alpha + i\sin\alpha, \alpha \in (0, 2\pi)$$

- (b) Compute (6)

$$z = \frac{\left\{ (1-i)^{10} (\sqrt{3}+i)^5 \right\}}{(-1-i\sqrt{3})^{10}}$$

- (c) Find the three roots of unity of the complex number $z = 1 + i$ and represent them in the complex plane. (6)

2. (a) For $a, b \in \mathbb{Z}/\{0\}$ define $a \sim b$ if and only if $ab > 0$. (6)

- (i) Prove that \sim defines an equivalence relation on \mathbb{Z} .

P.T.O.

(ii) What is the equivalence class of 5 ? What is the equivalence class of -5 ?

(b) Find the gcd (1800, 756). (6)

(c) Define $S : \mathbb{R} \rightarrow \mathbb{R}$ by $S(x) = x - \lfloor x \rfloor$. Is S is one to one ? Is it onto ? Explain. (6)

3. (a) Given natural numbers a and b , show that there are unique non – negative integers q and r with $0 \leq r < b$ such that $a = bq + r$. (6)

(b) Show that the open intervals $(1,3)$ and $(0,\infty)$ have the same cardinality. (6)

(c) If $ac \equiv bc \pmod{m}$ and $(c, m) = 1$ then $a \equiv b \pmod{m}$. (6)

4. (a) Determine the values of h and k such that the system

$$x_1 + hx_2 = 2$$

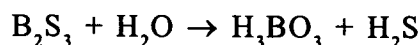
$$4x_1 + 8x_2 = k$$

has (i) no solution (ii) a unique solution (iii) many solutions (6½)

(b) Let $v_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$, $v_2 = \begin{bmatrix} -3 \\ 1 \\ 8 \end{bmatrix}$ and $y = \begin{bmatrix} h \\ -5 \\ -3 \end{bmatrix}$.

For what values(s) of h is y in the plane generated by v_1 and v_2 . (6½)

- (c) Balance the given chemical equation where Boron Sulphide reacts violently with water to form boric acid and hydrogen sulphide gas. The unbalanced equation



Here, for each compound, construct a vector that lists the number of atoms of boron sulphur, hydrogen and oxygen. (6½)

5. (a) Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^4$ be defined as

$$T(x_1, x_2) = (2x_2 - 3x_1, x_1 - 4x_2, 0, x_2).$$

- (i) Prove that T is a linear transformation.

- (ii) Find the standard matrix of T . (6½)

- (b) Let $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear transformation and let A be the standard matrix for T . Then prove that

- (i) T maps \mathbb{R}^n onto \mathbb{R}^m if and only if columns of A spans \mathbb{R}^m .

- (ii) T is one to one if and only if columns of A are linearly independent. (6½)

- (c) Find the basis for the column space and null space of the matrix

$$A = \begin{bmatrix} 4 & 5 & 9 & -2 \\ 6 & 5 & 1 & 12 \\ 3 & 4 & 8 & -3 \end{bmatrix} \quad (6½)$$

6. (a) (i) Define a subspace H of \mathbb{R}^n and its dimension too.

Is $H = \{(a, b, c, d) \mid c = a + 2b + 3d\}$ a subspace of \mathbb{R}^4 . Justify your answer. (6½)

- (b) Determine the dimension of the subspace H of \mathbb{R}^3 spanned by the vectors

$$v_1 = \begin{bmatrix} 2 \\ -8 \\ 6 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 3 \\ -7 \\ -1 \end{bmatrix} \quad \text{and} \quad v_3 = \begin{bmatrix} -1 \\ 6 \\ -7 \end{bmatrix} \quad (6\frac{1}{2})$$

- (c) Is $\lambda = 3$ an eigen value of the matrix $\begin{bmatrix} 1 & 2 & 2 \\ 3 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$? If so, find one corresponding eigen vector. (6 $\frac{1}{2}$)