[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1789 **FC-3** **Your Roll No.....**

Unique Paper Code : 32351102

Name of the Paper : C2 – Algebra

Name of the Course : **B.Sc. (H) Mathematics – I (CBCS)**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All **six** questions are compulsory.
3. Do any **two** parts from each question.

1. (a) Find the polar representation of the complex number (6)

$z = 1 + \cos\alpha + \sin\alpha,\ \alpha \in (0, 2\pi)$

(b) Compute (6)

$$z = \frac{\left((1-i)^{10}\left(\sqrt{3}+i\right)^{5}\right)}{\left(-1-i\sqrt{3}\right)^{10}}$$

(c) Find the three roots of unity of the complex number $z = 1 + i$ and represent them in the complex plane. (6)

2. (a) For $a, b \in Z/\{0\}$ define $a \sim b$ if and only if $ab > 0$. (6)

(i) Prove that $\sim$ defines an equivalence relation on Z.

P.T.O.

(ii) What is the equivalence class of 5 ? What is the equivalence class of -5 ?

(b) Find the gcd (1800, 756). (6)

(c) Define $S : IR \longrightarrow IR$ by $S(x) = x - \lfloor x \rfloor$. Is S is one to one ? Is it onto ? Explain. (6)

3. (a) Given natural numbers a and b, show that there are unique non – negative integers q and r with $0 \leq r < b$ such that $a = bq + r$. (6)

(b) Show that the open intervals (1,3) and $(0,\infty)$ have the same cardinality. (6)

(c) If $ac \equiv bc \pmod{m}$ and $(c, m) = 1$ then $a \equiv b \pmod{m}$. (6)

4. (a) Determine the values of h and k such that the system

$$x_1 + hx_2 = 2$$

$$4x_1 + 8x_2 = k$$

has (i) no solution (ii) a unique solution (iii) many solutions (6½)

(b) Let $v_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$, $v_2 = \begin{bmatrix} -3 \\ 1 \\ 8 \end{bmatrix}$ and $y = \begin{bmatrix} h \\ -5 \\ -3 \end{bmatrix}$.

For what values(s) of h is y in the plane generated by v_1 and v_2. (6½)

(c) Balance the given chemical equation where Boron Sulphide reacts violently with water to form boric acid and hydrogen sulphide gas. The unbalanced equation

$$B_2S_3 + H_2O \rightarrow H_3BO_3 + H_2S$$

Here, for each compound, construct a vector that lists the number of atoms of boron sulphur, hydrogen and oxygen. (6½)

5. (a) Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^4$ be defined as

$$T(x_1, x_2) = (2x_2 - 3x_1, x_1 - 4x_2, 0, x_2).$$

(i) Prove that T is a linear transformation.

(ii) Find the standard matrix of T. (6½)

(b) Let $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear transformation and let A be the standard matrix for T. Then prove that

(i) T maps $\mathbb{R}^n$ onto $\mathbb{R}^m$ if and only if columns of A spans $\mathbb{R}^m$.

(ii) T is one to one if and only if columns of A are linearly independent. (6½)

(c) Find the basis for the column space and null space of the matrix

$$A = \begin{bmatrix} 4 & 5 & 9 & -2 \\ 6 & 5 & 1 & 12 \\ 3 & 4 & 8 & -3 \end{bmatrix} \quad (6½)$$

6. (a) (i) Define a subspace H of $\mathbb{R}^n$ and its dimension too.

Is $H = \{(a, b, c, d\} \mid c = a + 2b + 3d\}$ a subspace of $\mathbb{R}^4$. Justify your answer. (6½)

(b) Determine the dimension of the subspace H of $\mathbb{R}^3$ spanned by the vectors

$$v_1 = \begin{bmatrix} 2 \\ -8 \\ 6 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 3 \\ -7 \\ -1 \end{bmatrix} \text{ and } v_3 = \begin{bmatrix} -1 \\ 6 \\ -7 \end{bmatrix} \qquad (6\frac{1}{2})$$

(c) Is $\lambda = 3$ an eigen value of the matrix $\begin{bmatrix} 1 & 2 & 2 \\ 3 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$? If so, find one corresponding eigen vector. (6½)