[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1112 **G** **Your Roll No...............**

Unique Paper Code : 235103

Name of the Paper : Analysis – I (MAHT-102)

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each questions.

1. (a) State and prove the Triangle Inequality and show that

$||a|-|b|| \le |a-b| \quad \forall a,b \in R$. (5)

(b) Given $S = \left\{1-\frac{1}{n} : n \in N\right\}$, show that Sup(S) = 1. (5)

(c) Let a > 0 and aS = [as: s ∈ S], Show that Sup(aS) = a Sap(S). (5)

2. (a) Show that arbitrary, intersection of a family of closed sets is a closed set. Is this result true for an arbitrary family of open sets ? Justify your answer. (5)

(b) Define Limit Point of a set. Show that the set of limit points of the set of rational numbers is R, the set of real numbers. (5)

(c) For non-empty bounded subsets A and B of R, show that

$$\inf(A + B) = \inf(A) + \inf(B).$$ (5)

3. (a) (i) Show that every convergent sequence is bounded but the converse is not true. (5)

(ii) Use the definition of the limit of a sequence to find the following limit :

$$\lim_{n\to\infty}\left(\frac{1}{n^2+1}\right).$$ (2½)

(b) (i) If (x_n) converges to x and (y_n) converges to y then show that $(x_n + y_n)$ converges to $(x+y)$. (5)

(ii) Give an example of two divergent sequences (x_n) and (y_n) such that $(x_n + y_n)$ converges. (2½)

(c) (i) Show that $\lim_{n\to\infty} n^{1/n} = 1$. (5)

(ii) Find the following limit :

$$\lim_{x\to\infty}\frac{n}{b^n},\quad b > 1$$ (2½)

4. (a) State and prove Cauchy Convergence criterion for sequence of real numbers. (5)

(b) Prove that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$. (5)

(c) Prove that the following sequence is a Cauchy sequence :

$$\left(1+\frac{1}{2!}+\cdots\frac{1}{n!}\right)$$ (5)

5. (a) (i) Find the sets of subsequential limits for the following sequences :

$(a_n) = (0,1,2,1,0,1,2,1,0,1,2,1,0,1,2,1,0 \ldots)$

$(b_n) = (2,1,1,0,2,1,1,0,2,1,0,1,2,1,1,0,2 \ldots)$. (2½)

(ii) Find the Limit inferior and Limit superior for the above defined sequences (a_n) and (b_n). (2½)

(b) Show that the following sequences are divergent :

(i) $a_n = (-1)^n$

(ii) $b_n = \sin\left(\frac{n\pi}{2}\right)$ (5)

(c) Let $x_1 = 8$, $x_{n+1} = \frac{x_n}{2} + 2$. Show that (x_n) is bounded and monotone. Also find its limit. (5)

6. (a) Give examples of the following series with justification :

(i) A divergent series $\sum a_n$ for which $\sum a_n^2$ converges.

(ii) A convergent series $\sum a_n$ for which $\sum a_n^2$ diverges. (5)

(b) Test the convergence of any two of the following series :

(i) $\sum \frac{\cos(n)+1}{3^n}$

(ii) $\sum \frac{1}{\sqrt{n+1}}$

(iii) $\sum \sqrt{n+1} - \sqrt{n}$ (5)

(c) State Ratio test for infinite series and show that $\sum \frac{n!}{3^n}$ diverges. (5)

7. (a) State Integral Test for series of real numbers and show that the series $\sum \frac{1}{n^p}$ is convergent if and only if $p > 1$. (5)

(b) Show that every absolutely convergent series is convergent but the converse is not true. (5)

(c) Examine the convergence of any two of the following series :

(i) $\sum \frac{(-1)^n}{n!}$

(ii) $\sum \frac{(-1)^{n+1}-(-1)^n}{n^2+1}$

(iii) $\sum \frac{(-1)^n}{2n+1}$ (5)