[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1788 **GC-3** **Your Roll No.....**

Unique Paper Code : 32351101

Name of the Paper : C 1 – Calculus

Name of the Course : **B.Sc. (Hons.) / Maths – I (CBCS)**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** the sections are compulsory.
3. **All** questions carry equal marks.
4. Use of non-programmable scientific calculator is allowed.

SECTION – I

Attempt any **four** *questions from Section I.*

1. If $\cos^{-1}\frac{y}{b} = \log\left(\frac{x}{n}\right)^n$ then show that

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0$$

2. Sketch the graph of

$f(x) = \frac{1}{3}x^3 - 9x + 2$ by finding intervals of increase and decrease, critical points, relative extrema and concavity for the given function.

P.T.O.

3. Find the horizontal asymptote to the graph of the function

$$f(x) = x^5\left[\sin\frac{1}{x} - \frac{1}{x} + \frac{1}{6x^3}\right]$$

4. It is projected that t years from now, the population of a certain country will be

$$P(t) = 50\,e^{0.02t} \text{ million}$$

(a) At what rate will the population be changing with respect to time 10 years from now.

(b) At what percentage rate will the population be changing with respect to time t years from now.

5. Sketch the graph of the curve in polar coordinates

$r^2 = 9\cos 2\theta$.

SECTION – II

Attempt any **four** *questions from Section – II.*

6. Find the reduction formula for $\int \sin^n x dx$ where n being positive integer and hence evaluate $\int_0^{\frac{\pi}{2}} \sin^n x dx$.

Further show that $\int_0^{\frac{\pi}{2}} \sin^n x dx = \int_0^{\frac{\pi}{2}} \cos^n x dx$.

7. Find the volume of the solid generated when the region enclosed by the curve $y = \sqrt{x}$, $y = 6 - x$ and $y = 0$ is revolved about x-axis.

8. Find the volume of the solid generated when the region enclosed by the curve $x = 2y - 2y^2$ and $x = 0$ is revolved about x-axis.

9. Find the arc length of the parametric curve $x = e^t \sin t$, $y = e^t \cos t$ for $0 \le t \le \frac{\pi}{2}$.

10. Find the area of the surface generated by revolving the curve $x = \sqrt{9 - y^2}$, $-2 \le y \le 2$, about y-axis.

SECTION – III

Attempt any **three** *questions from Section – III.*

11. Find the equation for a hyperbola passing through the origin with asymptotes

$$y = 2x + 1 \text{ and } y = -2x + 3.$$

12. Find the equation of the ellipse whose foci are (1, 2) and (1, 4) and whose minor axis is of length 2.

13. Describe the graph of the equation $x^2 - 4y^2 + 2x + 8y - 7 = 0$.

14. Trace the conic $x^2 + 2\sqrt{3}xy + 3y^2 + 2\sqrt{3x} - 2y = 0$ by rotating the coordinate axes to remove the xy term.

SECTION – IV

Attempt any **four** *questions from Section – IV.*

15. Find tangent vector and parametric equation of tangent line to the graph of the vector function

$$\vec{F}(t) = t^2\hat{i} + (\cos t)\hat{j} + (t^2 \cos t)\hat{k} \quad \text{at} \quad t = \frac{\pi}{2}.$$

16. A shell is fired with muzzle speed 150 m/s and angle of elevation 45° from a position 10 m above ground level. Where does the projectile hit the ground and with what speed ?

17. Find the tangential and normal components of acceleration of an object that moves along the parabolic path $y = 4x^2$ at the instant the speed is $\frac{ds}{dt} = 20$.

18. An object moves along the curve

$$r = \frac{1}{1-\cos\theta} \text{ and } \theta = t$$

Find its velocity and acceleration in terms of unit polar vectors u_r and u_θ.

19. Find the curvature and radius of curvature for a curve

$x = 3\cos t,\ y = 4\sin t,\ z = t$ at $t = \frac{\pi}{2}$.