

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1837

GC-3

Your Roll No.....

Unique Paper Code : 42351101

Name of the Paper : Mathematics – I : Calculus and Matrices (Course Code-235)

Name of the Course : **B.Sc. (Mathematical Sciences) / B.Sc. (Physical Sciences)**

Semester : I

Duration : 3 Hours

Maximum Marks : 75

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any two questions from each Section.

**SECTION I**

1. (a) Define basis of a vector space.

Is the set S,

$$S = \left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$$

of vectors constitute a basis for  $\mathbb{R}^3$  ? (6)

- (b) Define a linear Transformation. Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be the transformation denoting reflection about the line  $y = -x$ . Show that T is a linear transformation. Also find the standard matrix representing T. (6)

*P.T.O.*

2. (a) Define rank of a matrix. Find the rank of the following matrix by using elementary row operations.

$$\begin{bmatrix} 1 & 1 & 2 & 3 \\ 1 & 3 & 0 & 3 \\ 1 & -2 & -3 & 0 \\ 1 & 1 & 2 & 3 \end{bmatrix} \quad (6)$$

- (b) Let

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{pmatrix}$$

Find eigen values of the matrix A and eigen vector corresponding to one of them. (6)

3. (a) Define a subspace of a vector space. Let W be the set of all points inside and on the unit circle in the xy-plane. Is W a subspace of xy-plane? Justify. (6)

- (b) Solve the system of equations :

$$x + y + 3z = 1$$

$$2x + 3y - z = 3$$

$$5x + 7y + z = 7 \quad (6)$$

## SECTION II

4. (a) Sketch the graph of  $y = \frac{1}{2}x^2 - 3x + \frac{11}{2}$ . Mention the transformation used at each step. (6)
- (b) A certain culture of bacteria grows at a rate that is proportional to the number present. It is found that the number doubles in 4 hours. How may be expected at the end of 24 hours? (6)

(c) Find  $\frac{d^n y}{dx^n}$ , where

$$y = \sin(ax + b). \quad (6)$$

5. (a) Discuss the convergence of the sequences : (6)

$$(i) \left\langle \frac{\sin n}{n} \cdot \frac{n}{3n+1} \right\rangle \quad (ii) \left\langle 1 + \left(-\frac{1}{2}\right)^n \right\rangle \quad (6)$$

(b) Show that

$$u(x, t) = 4 \cos(2x + 2ct) + e^{x+ct},$$

$$\text{is a solution of the wave equation } \frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}. \quad (6)$$

$$(c) \text{ If } u = \frac{1}{\sqrt{x^2 + y^2 + z^2}}, \quad x^2 + y^2 + z^2 \neq 0$$

$$\text{Show that } \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0. \quad (6)$$

6. (a) If  $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$  show that

$$(1-x^2)y_{n+2} - (2n+3)xy_{n+1} - (n+1)^2 y_n = 0. \quad (6)$$

(b) Find the  $n^{\text{th}}$  Maclaurins polynomial for  $\frac{1}{1-x}$ . (6)

(c) Draw the level curve of  $f(x, y) = y^2 - x^2$  of height  $k = 1$ . (6)

## SECTION III

7. (a) Give the geometrical representation of difference of two complex numbers. (3½)

- (b) State Fundamental Theorem of Algebra. Also form an equation in lowest degree with real coefficients having  $2 + \sqrt{-3}$  and  $3 + \sqrt{-5}$  as two of its roots. (4)

8. (a) Solve the equation

$$z^4 + z^3 + z^2 + z + 1 = 0. \quad (4)$$

- (b) Show that

$$(1 + \cos\theta + i \sin\theta)^n + (1 + \cos\theta - i \sin\theta)^n = 2^{n+1} \cos^n \frac{\theta}{2} \cos \frac{n\theta}{2}. \quad (3\frac{1}{2})$$

9. (a) Find the equation of the circle described on the line joining the points  $(-1 - 3i)$  and  $(5 + 7i)$  as extremities of one of its diameters. (4)

- (b) Find the equation of the right bisector of the line joining the points  $z_1$  and  $z_2$ . (3½)