[This question paper contains 4 printed pages.]

1466-A *Your Roll No.* .....

B.A./B.Sc. (Hons.)/II **A**

MATHEMATICS – Unit VII

(Mechanics – I)

(Admissions of 2008 and before)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

All *Sections are compulsory.*
Attempt any **two** *parts from each Section.*

SECTION I

1. (a) Define equipollent force systems. Show that any force system in the fundamental plane may be reduced either to a single force or to a couple. (5)

(b) The ends A, B of a light rod AB are joined by light inextensible string AO, BO to a fixed point O, AO, BO being equal in length and at right angles to one another. Weights W_1 and W_2 are suspended from A and B. Show that the rod will take up an inclination θ to the horizontal, where

$$\tan\theta = \frac{W_1 \sim W_2}{W_1 + W_2}$$ (5)

P.T.O.

(c) A uniform square lamina rests in equilibrium in a vertical plane under gravity with two of its sides in contact with smooth pegs in the same horizontal line at a distance c apart. Show that the angle θ made up by a side of the square with the horizontal in a non-symmetrical position of equilibrium is given by

$$c(\sin\theta + \cos\theta) = a,$$

2a being the length of a side of the square. (5)

SECTION II

2. (a) If the density of any point of an arc of a uniform circular wire varies as its distance from the central radius, prove that the centre of mass is at a point on the central radius, midway between the chord and the arc. (4½)

(b) Two light rings can slide on a rough horizontal rod. The rings are connected by a light inextensible string of length a, to the mid-point of which is attached a weight W. Show that the greatest distance between the rings, consistent with the equilibrium of the system is

$$\mu a / \sqrt{1 + \mu^2}$$

where μ is the coefficient of friction between either ring and the rod. (4½)

(c) A uniform heavy rod of length $2l$ rests with its ends on a fixed smooth parabola with axis vertical and vertex downwards (latus rectum = 4a). Show that if $l > 2a$ there are three positions of equilibrium and that the horizontal position is then unstable, but that if $l < 2a$ the only position of equilibrium is horizontal which is stable. (4½)

SECTION III

3. (a) Define vector displacement of any particle of a body arising from a general infinitesimal displacement of the body. Prove that in any displacement of a rigid body, the amount of work done by equipollent force systems are equal. (5)

(b) Forces X, Y, Z act along the three lines given by the equations

$$y = 0,\ z = c;\quad z = 0,\ x = a;\quad x = 0,\ y = b;$$

prove that pitch of the equivalent Wrench is

$$(aYZ + bZX + cXY)/(X^2 + Y^2 + Z^2).$$

If the Wrench reduces to a single force, show that the line of action of the force must lie on the hyperboloid

$$(x - a)(y - b)(z - c) - xyz = 0 \qquad (5)$$

P.T.O.

(c) Two smooth planes intersect in a horizontal line and are inclined at the same angle α to the vertical. A uniform rod of weight W and length 2a is placed between them in a horizontal position making an angle θ with their line of intersection. Show that the horizontal couple required to maintain equilibrium is $Wa \cos\theta \cot\alpha$. (5)

SECTION IV

4. (a) Prove that in a homogeneous fluid at rest under gravity the difference between the pressures at two points is proportional to the difference of their depths. (4½)

(b) A rectangular area is immersed in a heavy liquid with two sides horizontal, and is divided by horizontal lines into strips on which the total thrusts are equal. Prove that if a, b, c are the breadths of three consecutive strips,

$a(a + b)(b - c) = c(b + c)(a - b)$ (4½)

(c) A plane lamina consists of a circular disc (radius a and centre O) from which a circular portion (radius a/2 and centre P) has been cut. The lamina is completely immersed in a homogeneous fluid with its plane vertical and P vertically below O. If OP is equal a/2 and the centre of pressure of the lamina is at O, prove that O is at a depth $\frac{11}{8}a$ below the surface of the fluid. (4½)