This question paper contains 8 printed pages.]

Your Roll No. ....

1465

B.Sc. (Hons.) / II A

MATHEMATICS – Paper VI

C++ Programming and Numerical Methods

(Admissions of 2009 and onwards)

Time : 3 Hours **Maximum Marks : 50**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.
Choice is given within the question.
Use of scientific calculator is allowed.

1. (a) Mark the following statements true or false. Justify your answer. 3

(i) The statement cin >> x >> y, requires the input values for x and y to appear on the same line.

(ii) The output of

```
if (7 < 16 ÷ 2)
cout <<"Hello";
cout <<"There";
```

is Hello There.

(iii) Let $x = 8$. After the statement y = x ++; executes, y is 9 and x is 8.

(b) Suppose the input is 6. What is the value of a after the following C++ statements executes ? Show execution. 2½

```
cin >>a;
if (a > 0)
switch (a)
{ case 1 :
        a = a + 3;
  case 3 :
        a ++;
        break;
  case 6 :
        a = a + 6;
  case 8 :
        a = a * 8;
        break;
default :
        a --;
}
else
        a = a + 2;
```

(c) Do a walk through to find the value assigned to e. Assume that all variables are properly declared. 1½

```
a = 3;
b = 4;
c = (a%b) * 6;
d = c/b;
e = (a + b + c + d)/4;
```

OR

1. (a) Write each of the following as C++ expression : 1½

(1) The character that represents 8.

(2) $\frac{b^2 - 4ac}{2a}$

(b) Correct the following code so that it prints the correct message : 1

```
if (score > = 60)
        cout << "You Pass." << endl;
else;
        cout <<"You Fail." << endl;
```

(c) What value is assigned to the variable a after the execution of the following statements ? Show execution. 4½

(i)
```
a = 17 < 4 * 3 + 5 || 8 * 2 = = 4 * 4 && !
(3 + 3 = = 6);
```

(ii)
```
a = static_cast < int > (2.5 + static_cast
< double> (15)/2);
```

2. (a) (i) How many times the following do ... while loop will be executed ? What is the output ? 2

```
x = 5;
y = 20;
do
    x = x + 2;
while (x > = y);
cout < < x < < "  " << y << endl;
```

(ii) How do you declare a pointer variable ? 1

(b) Suppose list is an array of six components of type int. What is stored in list after the following C++ statements executes ? 4

```
      list[0] = 5;
for   (i = 1; i < 6; i + +)
      {    list[i] = i * i + 5;
           if (i > 2)
           list[i] = 2 * list [i] – list [i – 1];
      }
```

OR

(a). Given : 3

enum currency Type {DOLLAR, POUND, FRANK, LIRA} currency;

Which of the following statements are invalid, explain why :

(i) Currency = DOLLAR;

(ii) cin >> currency;

(iii) Currency = static_cast <currency Type> (Currency + 1);

(b) Consider the following function : 2½

```
int test (int a, double b, char ch)
{     int x;
      if ('A' < = ch && ch < = 'R')
       return (2*a + static_cast <int> (b));
      else
       return (static_cast <int> (2*b) – a);
}
```

What is the output of C++ statement ?

cout << 2* test (6, 3.9, 'D') < < endl;

(c) Consider the following declarations : 1½

const int carTypes = 4;
const int colourTypes = 5;
int sales [car Types] [colour Types];

(i) How many components does array sales have ?

(ii) What is the number of rows in the array sales ?

(iii) What is the number of columns in the array sales ?

3. (a) Construct an algorithm for the method of false position. Use this algorithm to determine p_3, the third approximation to the location of the root of the equation $\cos x - x = 0$ on the interval (0, 1). **6**

(b) Let $A = \begin{bmatrix} 2 & 7 & 5 \\ 6 & 20 & 10 \\ 4 & 3 & 0 \end{bmatrix}$. **6**

Obtain the LU-decomposition of A and use it to solve the system $Ax = [0\ 4\ 1]^T$.

OR

(a) Define the order of convergence of an iterative method. Perform two iterations of the bisection method to determine p_2, the second approximation to the location of the root of the equation $x^3 + x^2 - 3x - 3 = 0$ on the interval (1, 2). Given the exact value of the root is $x = \sqrt{3}$, compute the absolute error in the approximations just obtained. 7

(b) Starting with the initial approximation vector $x^{(0)} = 0$ and relaxation parameter $w = 0.9$, perform two iterations of the SOR method to solve the following system of equations : 5

$$5x_1 + x_2 + 2x_3 = 10$$
$$-3x_1 + 9x_2 + 4x_3 = -14$$
$$x_1 + 2x_2 - 7x_3 = -33$$

4. (a) A thermodynamics student needs the temperature of saturated steam under a pressure of 6.3 Mega-Pascals (MPa). Estimate the temperature using linear interpolation from the data : 3

Pressure (MPa)	**Temperature (°C)**
6.0	275.64
7.0	285.88

(b) Construct the divided difference table for the following data set and then write out the Newton ' form of the interpolating polynomial : 5

x	−1	0	1	2
y	3	−1	−3	1

(c) Define the shift operator, E and the central difference operator δ. Prove that $E = 1 + \frac{1}{2}\delta^2 + \delta\sqrt{1 + \frac{1}{4}\delta^2}$ 4

OR

4. (a) If $x_0, x_1, \ldots, x_n$ are n + 1 distinct points and f is defined at $x_0, x_1, \ldots, x_n$, then show that there exists a unique Lagrange interpolating polynomial P of degree at most n_0. 4

(b) If $f(x) = e^{ax}$, show that $\Delta^n f(x) = (e^{an} - 1)^n e^{ax}$. 2

(c) The following data represents the function $f(x) = \cos(x + 1)$: 6

x	0.0	0.2	0.4	0.6
$f(x)$	0.5403	0.3624	0.1700	−0.0292

Construct the interpolating polynomial that fits the above data using the Gregory-Newton backward difference interpolation. Hence estimate f(0.5). Compare with the exact value.

5. (a) Use to the formula $f'(x_0) \approx \dfrac{f(x_0 + h) - f(x_0)}{h}$ 5

to approximate the derivative of $f(x) = 1 + x + x^3$ at $x_0 = 1$, taking h = 1, 0.1, 0.01 and 0.001. What is the order of approximation. Also demonstrate how the corresponding error varies as the step size h is cut by a factor of 10.

(b) Approximate the value of the integral $\int_1^2 \frac{1}{x}$ dx using the trapezoidal rule. Compare the error involved with the theoretical error bound. 4

(c) Apply Euler's method to approximate the solution of the initial value problem 3

$$\frac{dx}{dt} = tx^3 - x, 0 \leq t \leq 1, x(0) = 1$$

over the interval [0, 1], using four steps.

OR

(a) Use the second order central difference approximation for the second order derivative of $f(x) = \ln(x)$ at $x_0 = 2$, taking h = 1, .1, .01 and .001. Demonstrate how the corresponding error varies as the stepsize h is cut by a factor of 10. 6

(b) Approximate the value of the integral $\int_0^1 e^{-x}dx$ using the Simpson $3/8^{th}$ rule. 3

(c) Apply Euler's method to approximate the solution of the initial value problem 3

$$\frac{dx}{dt} = t^2 - 2x^2 - 1, 0 \leq t \leq 1, x(0) = 0$$

over the interval [0, 1] using four steps.