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9655

Your Roll No.

B.A. / B.Sc. (Hons.) / II **B**
MATHEMATICS – Unit VI
(Differential Equations – I)
(Admissions of 2008 and before)

Time : 2 Hours

Maximum Marks : 38

*(Write your Roll No. on the top immediately
on receipt of this question paper.)*

All the sections / questions are compulsory.

Section I

1. (a) Solve any two of the following :

(i) $\frac{dy}{dx} + y \cot x = 5 e^{\cos x}$ 2½

(ii) $x(4y dx + 2x dy) + y^3(3y dx + 5x dy) = 0$ 2½

(iii) $y p^2 - 2x p + y = 0$ 2½

[P.T.O.]

- (b) In a certain city the population gets doubled in 2 years and after 3 years the population is 20,000. Find the number of people initially living in the city. 4

Section II

2. (a) Attempt any **two** of the following :
- (i) Using method of variation of parameters,
 solve $\frac{d^2 y}{dx^2} + 4y = e^x$ 3
- (ii) Using method of undetermined coefficients,
 solve $\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} + y = x - e^x$ 3
- (iii) Solve $(D^2 + 1)y = x^2 \sin 2x$ 3
- (b) Prove that two solutions of the linear homogeneous second order differential equation
- $$a_0(x)\frac{d^2 y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)y = 0,$$
- $a_0(x) \neq 0, x \in [a, b]$
- $a_0(x), a_1(x), a_2(x)$ are continuous are linearly dependent if and only if their Wrenskian is identically zero. 4

Section III

3. Find the power series solution of any two of the following :

(i) $(2x + x^3) \frac{d^2 y}{dx^2} - \frac{dy}{dx} - 6xy = 0$ about $x = 0$ 5

(ii) $(x^2 + 1) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - xy = 0$ about $x = 0$ 5

(iii) $x \frac{d^2 y}{dx^2} + \frac{dy}{dx} + xy = 0$ about $x = 0$ 5

Section IV

4. (a) Solve any two of the following :

(i) $\frac{dx}{dt} + 5x + y = e^t$
 $\frac{dy}{dt} + 3y - x = e^{2t}$ 3

(ii) $\frac{yz}{y-z} dx = \frac{zx}{z-x} dy = \frac{xy}{x-y} dz$ 3

(iii) $a^2 y^2 z^2 dx + b^2 z^2 x^2 dy + c^2 x^2 y^2 dz = 0$ 3

- (b) Using Picard's method, find up to three successive approximations the solution of differential equation

$$\frac{dy}{dx} = 3e^x + 2y, y(0) = 0 \quad 3$$