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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **8812**

Unique Paper Code : **235302** **C**

Name of the Paper : **III-2—Numerical Methods and Programming**

Name of the Course : **B.Sc. (H) Mathematics Part II**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the *six* questions are compulsory.

Attempt any *two* parts from each question.

Marks are indicated against each question.

Use of Scientific Calculator is allowed.

1. (*a*) Derive rate of convergence of the Bisection method to obtain a root of $f(x) = 0$. Hence determine the minimum number of iterations n required by the Bisection method to converge within an absolute error tolerance of 0.01, starting with the initial interval (2, 3).

(*b*) Explain geometrically the method of false position to determine an approximation to a root of $f(x) = 0$. Using this method, perform 3 iterations to determine an approximation to a root of $x^4 - 18x^2 + 45 = 0$ lying in the interval (3, 4).

(*c*) Apply Secant method to $x^3 + x^2 - 3x - 3 = 0$ to determine an approximation to a root lying in the interval (1, 2). Iterate until the absolute error is less than 10^{-4}. 13

P.T.O.

2. (*a*) Using fixed point iteration method, perform 4 iterations to locate a fixed point of $g(x) = e^{-x}$, starting with $p_0 = 0$. What order of convergence do you expect ? Justify your answer.

(*b*) Write an algorithm for the Bisection Method to determine an approximation to a root of $f(x) = 0$. Use this method to determine the 3rd approximation to a root of $\cos(x) - x = 0$ lying in (0, 1).

(*c*) Obtain Newton's formula to determine $1/n$, where n is a natural number. Use it to obtain 1/37, starting with a suitable initial approximation. Do 3 iterations. 13

3. (*a*) Define a lower and an upper triangular matrix. Solve the system of equations :

$$x_1 + x_2 + 2x_3 = 3$$
$$-x_1 + 2x_3 = -1$$
$$3x_1 + 2x_2 - x_3 = 4$$

by obtaining an LU decomposition of the coefficient matrix A of the above system.

(*b*) Set up the Gauss-Jacobi iteration scheme to solve the system of equations :

$$4x_1 + x_2 + x_3 = 2$$
$$x_1 + 5x_2 + 2x_3 = -6$$
$$x_1 + 2x_2 + 3x_3 = -4$$

Take the initial approximation as $X^{(0)} = (0.5, -0.5, -0.5)$ and do two iterations.

(*c*) Set up the Gauss-Seidel iteration scheme to solve the system of equations :

$$6x_1 - 2x_2 + x_3 = 11$$
$$-2x_1 + 7x_2 + 2x_3 = 5$$
$$x_1 + 2x_2 - 5x_3 = -1$$

Take the initial approximation as $X^{(0)} = (1, 0, 0)$ and do three iterations. 12

4. (a) Given a function f defined at $n + 1$ distinct points $x_0, x_1, \ldots\ldots, x_n$, prove that there exists a unique polynomial $P_n(x)$ of degree at most n such that :

$$P_n(x_i) = f(x_i)\ ,\ i = 0, 1, 2, \ldots\ldots, n.$$

(b) Find the interpolating polynomial that fits the data :

x	0	1	2	4	5	6
$f(x)$	1	14	15	5	6	19

Hence interpolate at $x = 3.0$.

(c) Determine the step size h that can be used in the tabulation of a function $f(x)$, $a \le x \le b$, at equally spaced nodal points so that the truncation error of the quadratic interpolation is less than ε. Apply the result to find h for $f(x) = \sin(x)$ in the interval $\left[0, \frac{\pi}{4}\right]$ at equally spaced nodal points so that the truncation error of the quadratic interpolation is less than 5×10^{-8}. 13

5. (a) Define the backward difference operator ∇, central difference operator δ and averaging operator μ. Prove that :

(i) $\mu = \sqrt{\left(1 + \frac{1}{4}\delta^2\right)}$

(ii) $\nabla = -\frac{1}{2}\delta^2 + \delta\sqrt{\left(1 + \frac{1}{4}\delta^2\right)}$.

(*b*) Verify that the forward difference approximation :

$$f'(x_0) = \frac{1}{2h}\left(-3f(x_0) + 4f(x_0 + h) - f(x_0 + 2h)\right)$$

for the first order derivative provides the exact value of the derivative, regardless of the value of h, for the functions $f(x) = 1$, $f(x) = x$, $f(x) = x^2$ but not for the function $f(x) = x^3$. Deduce the order of the formula from this ?

(*c*) Using $f(x) = \ln(x)$ and $x_0 = 2$, demonstrate numerically (by taking $h = 0.1, 0.01, 0.001$) that the central difference approximation for the second derivative given by :

$$f''(x_0) = \left(f(x_0 - h) - 2f(x_0) + f(x_0 + h) - f(x_0 + h)\right)/h^2$$

is second order accurate. 12

6. (*a*) Define degree of precision of a quadrature rule. State Simpson's rule for the evaluation of $\int_a^b f(x)\,dx$ and verify that it has degree of precision 3.

(*b*) State Boole's rule for the evaluation of a definite integral and use it to approximate the value of $\int_1^2 \left(\frac{1}{x}\right) dx$.

(*c*) Apply Euler's Method to approximate the solution of the initial value problem :

$$\frac{dx}{dt} = e^t/x, \quad (0 \le t \le 2), \; x(0) = 1,$$

over the interval [0 , 2] using 5 steps. 12