[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 6610 **D** **Your Roll No...............**

Unique Paper Code : 235301

Name of the Course : **B.Sc. (Hons.) / Mathematics / Part – II**

Name of the Paper : III.1 Calculus II

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** sections are compulsory.
3. Attempt any **Five** questions from each section.

SECTION I

1. (a) Let f be the function defined by :

$$f(x,y) = \begin{cases} \dfrac{x - y^2}{x^2 + y^2}, & \text{for } (x,y) \neq (0,0) \\ 0, & \text{for } (x,y) = (0,0) \end{cases}$$

Is f continuous at (0,0) ? Justify your answer.

(b) Let $f(x,y) = x\ln(x + y^2)$ be any surface. Compute the slope of the tangent line to the graph of f at (e,0,e) in the direction parallel to

(i) the xz -plane, and (ii) the yz -plane. (3+2)

2. When two resistors with resistances P and Q ohms are connected in parallel, the combined resistance R satisfies

$$\frac{1}{R} = \frac{1}{P} + \frac{1}{Q}.$$

If P and Q are measured at 6 and 10 ohms, respectively, with errors no greater than 1%, what is the maximum percentage error in the computation of R ? (5)

3. (a) If $z = xy + f(x^2 + y^2)$, then compute : $y\frac{\partial z}{\partial x} - x\frac{\partial z}{\partial y}$.

(b) Find $\frac{\partial w}{\partial r}$, where $w = e^{2x-y+3z^2}$ and $x = r + s - t$, $y = 2r - 3s$, $z = \cos rst$.

(2+3)

4. (a) Find equations for the tangent plane and the normal line at the point $(1, -1, 2)$ on the surface $x^2y + y^2z + z^2x = 5$.

(b) Find the directional derivative of $f(x, y) = e^{x^2y^2}$ at $P(1, -1)$ in the direction towards $Q(2,3)$. (3+2)

5. Find the point on the plane $x + 2y + z = 5$ that is closest to the point $(0, 3, 4)$. (5)

6. Use the method of Lagrange multipliers to find the largest and smallest values of $f(x, y) = 16 - x^2 - y^2$ subject to constraint $x + 2y = 6$, $x \geq 0$, and $y \geq 0$. (5)

SECTION II

7. Find the volume of the solid bounded above by the plane $z = y$ and below in the xy-plane by the part of the disk $x^2 + y^2 \leq 1$ in the first quadrant, using a double integral. (5)

8. Sketch the region of integration and compute the integral

$$\int_0^1 \int_x^{2x} e^{y-x} \, dy dx$$

with the order of integration reversed. (5)

9. Use polar co-ordinates to evaluate.

$$\int_0^2 \int_y^{\sqrt{8-y^2}} \frac{1}{\sqrt{1 + x^2 + y^2}} dA$$ (5)

10. Find the volume of the solid in the first octant that is bounded by the cylinder $x^2 + y^2 = 2y$, the half cone $z = \sqrt{x^2 + y^2}$, and the xy-plane. (5)

11. Evaluate

$$\iiint_D \frac{dxdydz}{\sqrt{x^2 + y^2 + z^2}}$$

where D is the solid sphere $x^2 + y^2 + z^2 \le 3$. (5)

12. Use a suitable change of variables to find the area of the region R bounded by the hyperbolas $xy = 1$ and $xy = 4$, and the lines $y = x$ and $y = 4x$. (5)

SECTION III

13. Evaluate

$$\int_C 5xydx + 10yzdy + zdz$$

where the path C consists of the parabolic arc $x = y^2$ from (0,0,0,) to (1,1,0) followed by the line segment given by $x = 1$, $y = 1$, $0 \le z \le 1$. (5)

14. Verify that the line integral

$$\int_C (3x^2 + 2x + y^2)dx + (2xy + y^3)dy$$

where C is any path from (0,0) to (1,1) is independent of path and then find its value. (5)

15. State Green's theorem for a simply connected region in R^2. Use Green's theorem to find the work done when an object moves in the force field

$\vec{F}(x, y) = y^2\,\vec{i} + x^2\,\vec{j}$ counter clockwise around the circular path $x^2 + y^2 = 2$.

(5)

16. Find the flux of the vector field $\vec{F}(x,y) = z\vec{i} + x\vec{j} + (y+z)\vec{k}$ through the parametrized surface

$$\vec{R}(u,v) = (uv)\vec{i} + (u-v)\vec{j} + (2u+v)\vec{k}$$

over the triangular region D in the uv-plane that is bounded by $u = 0$, $v = 0$ and $u + v = 1$. (5)

17. State Stoke's theorem. Use Stoke's theorem to evaluate the line integral

$$\oint_C (3ydx + 2zdy - 5xdz)$$

where C is the intersection of the xy-plane and the hemisphere

$z = \sqrt{1 - x^2 - y^2}$, traversed counter clockwise as viewed from above. (5)

18. Use Divergence theorem to evaluate

$$\iint_S \left(\vec{F}.\vec{N}\right) dS$$

where $\vec{F}(x,y,z) = \left(x^5 + 10xy^2z^2\right)\vec{i} + \left(y^5 + 10yx^2z^2\right)\vec{j} + \left(z^5 + 10zx^2y^2\right)\vec{k}$ and S is the closed hemispherical surface $z = \sqrt{1 - x^2 - y^2}$ together with the disk $x^2 + y^2 \le 1$ in the xy-plane. (5)

19. State and prove the Divergence theorem. (5)