[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 5011 **D** **Your Roll No...............**

Unique Paper Code : 236362

Name of the Course : **B.Sc. Mathematical Sciences**

Name of the Paper : OPERATIONAL RESEARCH – III : MATHEMATICAL PROGRAMMING

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **five** questions.
3. **All** questions carry equal marks.
4. Statistical Tables can be used.
5. Calculators are allowed.

1. (a) Describe the various types of Integer Programming problem, also mention the various methods for solving them. Describe any one. (7)

   (b) Solve the following Pure Integer programming Problem. (8)

   Maximize $Z = x_1 + x_2$

   s.t. (i) $3x_1 + 2x_2 \leq 5$ (ii) $x_2 \leq 2$

   and $x_1, x_2 \geq 0$ and are integers

   The Optimal non integer table for the above problem is

C_B	B	b	x_1	x_2	S_1	S_2
1	x_1	1/3	1	0	1/3	−2/3
1	x_2	2	0	1	0	1
Max Z=7/2,		Z_j-C_j	0	0	1/3	1/3

2. (a) Consider the following un-constrained optimization problem;

$$\max f(x) = x^3 + 2x - 2x^2 - 0.25x^4$$

Apply Bisection method (one dimensional search procedure) to approximately solve the problem. Use an error tolerance $\varepsilon = 0.04$ and initial lower bound 0 and upper bound 2.4. (apply maximum 4 iterations) (10)

(b) Describe Zero-One Integer programming, showcase it with the help of an example. (5)

3. (a) Let f(x) be differentiable in its domain. If f(x) is defined on an open convex set S then show that f(x) is convex if and only if

$$f(x_2) - f(x_1) \geq (x_2 - x_1)^T \nabla f(x_1)$$

for all $x_1, x_2 \in S$. (8)

(b) A company manufactures two products if it charges a price p_i for the product i, it can sell q_i units of product i, where $q_1 = 60 - 3p_1 + p_2$ and $q_2 = 80 - 2p_2 + p_1$. It cost 25 rupees to produce one unit of product 1 and 72 rupees to produce a unit of product 2. How many units of each product should be produced to maximize profits ? (7)

4. (a) Derive the necessary and sufficient condition for optimality for a Generalized Multivariable non-linear optimization problem with one equality constraint using **Lagrange Multiplier** method. (6)

(b) Use dichotomous search method; find the maximum of the following problem;

$$f(x) = \begin{cases} 4x, & 0 \le x \le 2 \\ 4-x & 2 \le x \le 4 \end{cases}$$

Use error tolerance 0.05 and attempt maximum 4 iterations. (9)

5. Find the optimum value of the objective function when separately subject to the following two sets of constraints using Kuhn Tucker conditions

Maximize $Z = 10x_1 - x_1^2 + 10x_2 - x_2^2$

Subject to the constraints

(i) $x_1 + x_2 \le 14$ (ii) $x_1 + x_2 \le 8$

$-x_1 + x_2 \le 6$ $-x_1 + x_2 \le 5$

$x_1, x_2 \ge 0$ $x_1, x_2 \ge 0$

Hence show the various conditions for the problems to be feasible. (15)

6. Use the Wolfe's modified simplex method to solve the **Quadratic programming problem :** (15)

Maximize $Z = 2x_1 + x_2 - x_1^2$

Subject to the constraints

(i) $2x_1 + 3x_2 \le 6,$ (ii) $2x_1 + x_2 \le 4$

$x_1, x_2 \ge 0$

7. (a) Define concave and convex function. Use definition to show that $f(x) = x^2$, is convex. (5)

(b) Determine the extreme points of the following function

$$f(x_1, x_2) = x_1^3 + x_2^3 - 3x_1x_2 \quad (5)$$

(c) Describe the gradient method for solving multivariate unconstrained optimization problem. (5)