[This question paper contains 4 printed pages.]

| Sr. No. of Question Paper | $: 5951 \quad$ D |
| :--- | :--- | :--- |
| Unique Paper Code | $: 235161$ |
| Name of the Course | $:$ B.Sc. (H) and B.Sc. (Prog............ |
| Name of the Paper | $:$ MATHEMATICS AND STATISTICS / MACT-303 |
| Semester | $:$ III/I |
| Duration $: 3$ Hours |  |

## Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. There are three Sections in this question paper.
3. Attempt any two questions from each Section.
4. Students are allowed to use simple calculators.

## SECTION - I

1. (a) Out of 450 students in a college, 193 know German language and 200 know French. 80 know neither. Illustrate this fact by a Venn diagram and find out how many know both.
(b) Examine the continuity of the following function at $x=0$

$$
\mathrm{f}(\mathrm{x})=\left\{\begin{array}{cc}
\mathrm{x}-1 & \mathrm{x}<0  \tag{5.5}\\
1 & \mathrm{x}=0 \\
2 \mathrm{x} & \mathrm{x}>0
\end{array}\right.
$$

(c) If $y=a+b x^{2}$, prove that $x \frac{d^{2} y}{d x^{2}}=\frac{d y}{d x}$.
2. (a) The size of a slowly growing bacterial culture is approximately given by

$$
N(t)=N_{0}+10 t+2 t^{3}
$$

Where $N_{0}$ is the size when $t=0$ and time $t$ is measured in hours. Find the average rate of growth during the time interval $t=1$ to $t=5$.
(b) Write down the first five terms of the sequence given by the recursion formula :

$$
\begin{equation*}
a_{1}=1, a_{2}=-1, a_{n+2}=\frac{a_{n+1}}{a_{n}} \tag{5}
\end{equation*}
$$

(c) Draw the graph of $y=\sin x, 0 \leq x \leq 2 \pi$.
3. (a) Find $\frac{d^{2} y}{d x^{2}}$ if
(i) $y=5 x^{4}+e^{x}+\sin x$
(ii) $y=\tan ^{2} x+\frac{3}{x^{2}}$
(b) Examine the nature of the roots of the equation $3 x^{2}-8 x+4=0$.
(c) Integrate the following:
(i) $\int \frac{x}{x^{2}+1} d x$
(ii) $\int\left(x+\frac{1}{x^{2}}\right)^{2} d x$
4. (a) Evaluate:

$$
\begin{equation*}
\int_{1}^{2}\left(2 x^{3}+x^{2}-4\right) d x \tag{6}
\end{equation*}
$$

(b) If the distance $S$ covered by a particle in $t$ seconds is given by

$$
S=a e^{t}+b e^{-t}
$$

Find the velocity and show that the acceleration at time $t$ is equal to the distance travelled by it up to time $t$.
(c) Find the sum of the infinite series

$$
\begin{equation*}
1+\frac{1}{2}+\frac{1}{2^{2}}+\frac{1}{2^{3}}+\ldots \tag{4.5}
\end{equation*}
$$

## SECTION - II

5. (a) If $\mathrm{A}=\left(\begin{array}{rr}6 & 1 \\ 7 & -4\end{array}\right)$ and $\mathrm{B}=\left(\begin{array}{rr}10 & -2 \\ -4 & 11\end{array}\right)$

Find the matrix $5 \mathrm{~A}-2 \mathrm{~B}$.
(b) Find the value of $x$ such that

$$
\left(\begin{array}{lll}
1 & 1 & \mathrm{x}
\end{array}\right)\left(\begin{array}{lll}
1 & 0 & 2  \tag{6}\\
0 & 2 & 1 \\
2 & 1 & 0
\end{array}\right)\left(\begin{array}{l}
1 \\
1 \\
1
\end{array}\right)=0
$$

6. (a) Find the values of $x, y, z$ and $w$ which satisfy the matrix equation:

$$
3\left(\begin{array}{cc}
x & y  \tag{5}\\
z & w
\end{array}\right)=\left(\begin{array}{cc}
x & 6 \\
-1 & 2 w
\end{array}\right)+\left(\begin{array}{cc}
4 & x+y \\
z+w & 3
\end{array}\right)
$$

(b) Find the image of the point $(5,3)$ under the following transformations using matrix multiplication.
(i) Dilation by a scale factor 2 .
(ii) Reflection in x -axis.
(iii) Rotation through an angle $45^{\circ}$ in the counter clockwise direction.
7. (a) If $\mathrm{A}=\left(\begin{array}{ll}1 & 3 \\ 5 & 2\end{array}\right) \quad \mathrm{B}=\left(\begin{array}{cc}2 & 3 \\ -1 & 4\end{array}\right)$

Calculate $\mathrm{AB}+\mathrm{BA}$.
(b) Find $A=\left(\begin{array}{rr}1 & 4 \\ 4 & -2 \\ 2 & 6\end{array}\right), \quad B=\left(\begin{array}{rr}-1 & -2 \\ 4 & 5 \\ 3 & 1\end{array}\right)$

Find matrix $X$ such that $A+2 B-3 X=0$.
SECTION - III
8. (a) Find the Arithmetic Mean of the following frequency distribution.

| Class Interval | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ | $50-60$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 4 | 10 | 8 | 20 | 15 | 12 |

(b) The following frequency table gives the ages (in years) of a group of 50 children invited to a birthday party. Find the standard deviation of the distribution.

| Age (in years) | $5-7$ | $7-9$ | $9-11$ | $11-13$ | $13-15$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 16 | 13 | 10 | 6 | 5 |

9. (a) Two random variables have the least squares regression lines $3 x+2 y=26$ and $6 x+y=31$. Find the mean values and the coefficient of correlation.
(b) The probability that a student passes in physics is 0.8 and the probability he passes mathematics is 0.7 . if 0.6 is the probability that he will pass in both examination then what is the probability that he will pass in atleast one of them.
10. (a) A die (unbiased) is rolled. A person gets Rs. 10 , if he gets an even number and losses Rs. 5 for an odd number. What is his expectation?
(b) An oil exploration firm finds that $5 \%$ of the test wells it drills, yields a deposite of natural gas if the firm drills six wells, use Poisson distribution to find the probability that
(i) Exactly two wells
(ii) Atleast one well
yield gas? $\left(\mathrm{e}^{-0.3}=0.7408\right)$
11. (a) The heights of plants of a certain species are normally distributed, the mean height being 30 cm and the standard deviation being 5 cm . What proportion of plants are greater than 40 cm in height?
(b) A certain stimulus administered to each of 12 patients resulted in the following increase in B.P.

$$
5,2,8,-1,3,0,-2,1,5,0,4,6
$$

Can it be concluded that the stimulus will in general be accompanied by an increase in blood pressure. $\left(\mathrm{t}_{0.05}\right.$ at 11 d.f. $\left.=2.201\right)$

