[This question paper contains 2 printed pages.]

Sr. No. of Question Paper : 1457 **F-7** **Your Roll No...............**

Unique Paper Code : 2351302

Name of the Paper : Analysis – II (Real Functions)

Name of the Course : **B.Sc. (H) Maths – II (Erstwhile FYUP)**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on the receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **three** parts from each question.

1. (a) Let $A \subseteq R$ and $c \in R$ be a cluster point of A and $f: A \to R$, then define limit of function f at 'c'. Also show that, f can have only one limit at 'c'. (5)

   (b) Let c E R. Use $\varepsilon - \delta$ definition to show that $\lim\limits_{x \to c} x^2 = c^2$. (5)

   (c) State and prove Sequential Criterion of Limits. (5)

   (d) Show that $\lim\limits_{x \to 0} \sin\left(\frac{1}{x}\right)$ does not exist, where as $\lim\limits_{x \to 0} x \sin\left(\frac{1}{x}\right) = 0$. (5)

2. (a) Let $A \subseteq R$, let f and g be functions on A to R and let $c \in R$ be a cluster point of A.

   Show that if $\lim\limits_{x \to c} f(x) = L$ and $\lim\limits_{x \to c} g(x) = M$ then $\lim\limits_{x \to c} (fg)(x) = LM$. (5)

   (b) Let $f(x) = e^{1/x}$ for $x \neq 0$, then find $\lim\limits_{x \to 0-} f(x)$ and $\lim\limits_{x \to 0+} f(x)$. (5)

(c) Show that $\varphi(x) = 1/x$ is continuous on $A = \{x \in R: x > 0\}$. (5)

(d) Let A = R and let f be the Dirichlet's function defined by

$$f(x) = \begin{cases} 1, & \text{for } x \text{ rational} \\ -1, & \text{for } x \text{ irrational} \end{cases}$$

Show that f is discontinuous at every point of R. (5)

3. (a) Let $A \subseteq R$ and $f : A \to R$. Show that if f is continuous at $c \in A$ then $|f|$ is continuous at c. Is the converse true ? Justify your answer. (5)

(b) Let f, g be continuous from R to R and suppose that f(r) = g(r) for all rational numbers r. Show that f(x) = g(x) for all $x \in R$. (5)

(c) Let f be a continuous real valued function defined on [a, b]. Show that f is a bounded on [a,b]. (5)

(d) Suppose that f is a real valued continuous function on R and that f(a)f(b) < 0 for some a, $b \in R$. Prove that there exists x between a and b such that f(x) = 0. Prove that $x2^x = 1$ for some x in (0,1). (5)

4. (a) Let I be a closed and bounded interval and let $f : I \to R$ be continuous on I. Then prove that f is uniformly continuous on I. (5)

(b) Show that the function $f(x) = x^2$ is not uniformly continuous on R. (5)

(c) If $f : I \to R$ has a derivative at $c \in I$, then show that f is continuous at c. Is the converse true? Justify your answer. (5)

(d) Let $f : I \to R$ be differentiable on the interval I. Prove that f is decreasing on I if and only if $f'(x) \leq 0$ for all $x \in I$. (5)

5. (a) State and prove Mean Value Theorem. (5)

(b) Show that $|\sin x - \sin y| < |x - y| \quad \forall x,y \in R$. (5)

(c) For the function $f : R \to R$ defined as $f(x) = x^2 - 3x + 5$, find the points of relative extrema. Also, find the intervals on which the function is increasing, and those on which it is decreasing. (5)

(d) Obtain Maclaurin's series expansion for the function $f(x) = \sin x$, $x \in R$. (5)

(400)