This question paper contains **4+1** printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **22**

Unique Paper Code : **236362** **G**

Name of the Paper : **Operational Research—III : Mathematical Programming**

Name of the Course : **B.Sc. Mathematical Sciences**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *Five* questions.

Simple calculator is allowed.

1. (*a*) Define convex function and by using the definition, show that x^2 is a Convex function. 3

(*b*) Examine whether the following functions are convex or concave and find the point of optima : 3+3

(*i*) $f(x_1, x_2, x_3) = x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1 x_2 + 6x_1 x_3 - 5x_2 x_3$

(*ii*) $f(x_1, x_2) = 2x_1^2 + 6x_2^2 - 6x_1 x_2$

(*c*) Find the maximum of the function :

$$f(x) = \begin{cases} 4x & 0 \le x \le 2 \\ 4-x & 2 \le x \le 4 \end{cases}$$

by using Dichotomous Search Procedure. Perform maximum three iterations and use delta ($\Delta = 0.1$). 6

P.T.O.

2. (*a*) Solve the following integer programming problem by using Branch and Bound method :

Max. $Z = 8x_1 + 5x_2$

Subject to

$$x_1 + x_2 <= 6$$

$$9x_1 + 5x_2 <= 45$$

$$x_1, x_2 >= 0$$

x_1 and x_2 are integers. 11

(*b*) A company manufactures two products A and B. It takes 30 minutes to process one unit of product A and 15 minutes for each unit of B and the maximum machine time available is 35 hours per week. Products A and B require 2 kgs and 3 kgs of raw material per unit respectively. The available quantity of raw material is envisaged to be 180 kgs per week.

The Products A and B which have unlimited market potential sell for Rs. 200 and Rs. 500 per unit respectively. If the manufacturing cost for products A and B are $2x^2$ and $3y^2$ respectively, where

x = Quantity of product A to be produced and

y = Quantity of product B to be produced.

Formulate it as a Non-linear programming problem to determine how many units of each product should be produced per week. 4

3. (*a*) Let $f(x)$ be a convex function on a convex set S. Then prove that the set of points is S at which $f(x)$ takes on its global minimum is a convex set. 7

(*b*) Solve the following Non-linear programming problem by using Lagrangian Multiplier Technique : 8

Optimize $Z = 6x_1^2 + 5x_2^2$

Subject to

$$x_1 + 5x_2 = 3$$

$$x_1, x_2 >= 0$$

4. (*a*) Solve the following mixed Integer Programming Problem by using Gomory's Cutting Plane method : 10

Max. $Z = 4x_1 + 3x_2$

Subject to

$$2x_1 + x_2 <= 4$$

$$3x_1 + 5x_2 <= 15$$

$$2x_1 + 5x_2 <= 10$$

$$x_1, x_2 >= 0$$

x_2 integer.

(*b*) Consider the following Constrained Non-Linear Programming Problem :

Max. $f(X)$

Subject to $g(x) <= C,\ x >= 0$

where $x = (x_1, x_2, ..., x_n)$.

Develop the Kuhn Tucker necessary conditions for the optimal solution of the given problem. Also state the sufficient conditions for the above problem. 5

5. Solve the following QPP (Quadratic Programming Problem) by using Wolfe's method : 15

Max. $Z = 2x_1 + 3x_2 - 2x_1^2$

Subject to

$$x_1 + 4x_2 <= 4$$

$$x_1 + x_2 <= 2$$

$$x_1, x_2 >= 0.$$

6. (*a*) Use KKT conditions to solve the following Non-linear programming problem : 9

Max. $Z = 4x_1 + 6x_2 - x_1^2 - x_2^2 - x_3^2$

Subject to

$$2x_1 + 3x_2 <= 12$$

$$x_1 + x_2 <= 2$$

$$x_1, x_2, x_3 >= 0$$

22

(*b*) Find the maximum of the function :

$$f(x) = \begin{cases} 3x & 0 \le x \le 2 \\ \dfrac{20 - x}{3} & 2 \le x \le 3 \end{cases}$$

by using Golden Section Search method. Perform maximum three iterations and use error tolerance epsilon ($\varepsilon = 0.1$).

6

7. (*a*) Apply maximum two iterations of Gradient Search Steepest Ascent method to solve the following problem :

8

$$\text{Max } f(x) = 2x_1x_2 + x_2 - x_1^2 - 2x_2^2.$$

Start with the initial trial solution (1, 1) and take error tolerance epsilon ($\varepsilon = 0.25$).

(*b*) Discuss Jacobian method to solve a constrained Non-linear optimization problem with equality constraint.

7