This question paper contains 4 printed pages.]

9663 *Your Roll No.* ....

B.A./B.Sc. (Hons.) / III B

MATHEMATICS – Paper XIV

(Mechanics-II)

(Admissions of 2008 and before)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

*Attempt **all** questions, selecting **two** parts of each question. All questions carry equal marks.*

1. (a) Derive expression for radial and transverse components of velocity and acceleration of a particle moving along a given curve in a plane.

(b) A bead of mass m slides on a smooth wire in the form of a parabola with its axis vertical and vertex downward. If the bead starts from rest at the end of latus rectum (of length $4p$), find the speed with which it passes through the vertex. Find also the reaction of the wire on the bead at this point.

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(c) An automobile travels round a circle of radius r. If h is the height of the centre of gravity above the ground and $2b$ the width between the wheels, show that it will overturn if the speed exceeds $\sqrt{grb/h}$ assuming that no side-slipping takes place.

2. (a) A particle moving in a straight line is subject to retardation of amount kv^n per unit mass, where v is the speed at time t. Show that if $n < 1$, the particle will come to rest at a distance $\frac{u^{2-n}}{k(2-n)}$ from the point of projection at time $t = \frac{u^{1-n}}{k(1-n)}$ where u is the initial speed. Discuss the case when :
(i) $1 < n < 2$; and (ii) $n > 2$.

(b) A particle of mass m moves on a straight line under the influence of a force directed towards the origin O on the line and proportional to the distance from origin O. The force at unit distance is of magnitude mk^2. The particle passes O with velocity U. If x is its co-ordinate at time t and V the velocity at any instant, show that
$V^2 + k^2x^2 = U^2$.

(c) A gun is mounted on a hill of height h above a level plane. Show that if the resistance of the air is neglected, the greatest horizontal range for a given initial velocity v is obtained by flying at an angle of elevation $\theta : \operatorname{cosec}^2 \theta = R\,(1 + gh / v^2)$.

3. (a) Define the central orbit and derive its differential equation.

(b) A particle of mass m describes an elliptical orbit of semi-major-axis a under a force $\frac{m\mu}{r^2}$ directed to a focus. Prove that the time average of square of the speed is $\frac{1}{T}\int\frac{dt}{r} = \frac{1}{a}$.

(c) A particle of mass m moves in a central field of attractive force of which the intensity is $mkr^{-2}\, e^{-r^2}$ where k is constant. Prove that the circular orbit of radius r is stable if and only if $r^2 < 1/2$.

4. (a) Derive the equation of momental ellipsoid. If $G(\alpha, \beta,)$ is centre of mass of a plane lamina of mass M in Oxy system, and GX and GY are principle axes then prove that the product of inertia of lamina with respect to Ox and Oy is $M\alpha\beta$.

(b) Three uniform rods each of mass m form an equilateral triangle of side 2a. The triangle is suspended from one corner. Find the length of the equivalent simple pendulum for oscillation under gravity when the triangle oscillates in its own plane.

(c) A body turns about a fixed point. Prove that angle between angular velocity vector and its angular momentum vector about the fixed point is always acute. Show that if principle moment of inertia A,B,C are all different then L vanishes only if the body is turning about principle axes.