This question paper contains 4+1 printed pages]

Your Roll No. ....

9667

B.A./B.Sc. (Hons.)/III **B**

MATHEMATICS : Paper XVII and XVIII (*ii*)

(Boolean Algebra)

Time : 2 Hours *Maximum Marks* : 38

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question except

Q. No. 3 and any *one* part of Q. No. 3.

All parts of every question carry equal marks.

1. (*a*) Give an example of a sublattice of a lattice L which is not convex. Prove that the intersection of an ideal and a dual ideal of L, if non-empty, is a convex sublattice of L. 5

(*b*) Prove that every homomorphic image of a relatively complemented lattice is relatively complemented. 5

(c) (i) Let M be a lattice with least element. Prove that the kernel of a homomorphism from a lattice L onto M is an ideal of L. 2½

(ii) Let C be a congruence relation on a lattice L and let L/C be the quotient lattice of L by C. Prove that there exists a natural homomorphism of L onto L/C. What is the kernel of this homomorphism, if L has the least element ? 2½

2. (a) If a, b are any elements of a modular lattice, then prove that $[a \wedge b, a] \cong [b, a \vee b]$. 5

(b) (i) Prove that a lattice of length two is modular. 2½

(ii) Prove that the cardinal product of two modular lattices is modular. 2½

(c) (i) Define a distributive lattice. Give an example of a modular lattice which is not distributive. $2\frac{1}{2}$

(ii) Let L be a distributive lattice and $a, b \in L$.

Show that $X = \{x \in L : x \wedge a = x \wedge b\}$ is an ideal of L. $2\frac{1}{2}$

3. (a) If L is a Boolean algebra, then prove that any ideal of L equals the kernel corresponding to one and only one congruence relation on L. 8

(b) (i) Prove that in a Boolean algebra :

$(a')' = a$, $(a \vee b)' = a' \wedge b'$

and $(a \wedge b)' = a' \vee b'$.

Hence show that a non-empty subset of a Boolean Algebra L is a subalgebra of L if it is closed under complementation and one of the two binary operations of meet and join. 4

(*ii*) Find the conjunctive normal form of the. Boolean function $(x \wedge (y' \vee z)) \vee z'$ and hence find its disjunctive normal form from it. 4

4. (*a*) Simplify the Boolean function and draw the circuit : 5

$$(a' \wedge b' \wedge c) \vee (a' \wedge b \wedge c') \vee (a' \wedge b \wedge c)$$

$$\vee (a \wedge b' \wedge c) \vee (a \wedge b \wedge c).$$

(*b*) Show that the Boolean function $(a' \wedge b' \wedge c) \vee (a \wedge b' \wedge c')$ represents a circuit using five switches. If $(a' \wedge b' \wedge c')$ and $(a \wedge b' \wedge c)$ happen to be don't care conditions, show that the circuit can be represented by only one switch. 5

(*c*) Draw the bridge and series parallel circuit for the Boolean function : 5

$$(a \wedge b) \vee [a \wedge e \wedge (d \vee b')] \wedge [c \wedge (d \vee b')] \wedge (c \wedge e \wedge b).$$

Find the Boolean expression (in CN form) that defines a function given by :

x	y	z	f
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1