[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 6616 **D** **Your Roll No.....**

Unique Paper Code : 235501

Name of the Course : **B.Sc. (Hons.) Mathematics**

Name of the Paper : Differential Equation and Mathematical Modeling III [Paper 5.1]

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for the Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **three** parts from each question.

1. (a) Using Laplace transform, solve the system of equations

$x' + 2y' + x = 0$

$x' - y' + y = 0 \qquad x(0) = 0, \quad y(0) = 1$ (6)

(b) (i) Show that the function $f(t) = \sin(e^{t^2})$ is of exponential order as $t \to \infty$ but that its derivative is not. (3)

(ii) Show that

$$\mathcal{L}\{t \cos h(kt)\} = \frac{s^2 + k^2}{(s^2 - k^2)^2}$$ (3)

(c) Determine the radius of convergence of a series solution of

$(x^2 + 9)y'' + xy' + x^2y = 0 \quad \ldots \quad (1)$

in powers of x and in powers of $(x - 4)$ by finding the singularities of (1). (6)

P.T.O.

(d) Find two linearly independent Frobenius series solution of

$4xy'' + 2y' + y = 0$ (6)

2. (a) Using Monte Carlo Simulation, write an algorithm to calculate an approximation to π by considering the number of random points selected inside the quarter circle

$$Q: x^2 + y^2 = 1, \quad x \geq 0, \quad y \geq 0$$

where the quarter circle is taken to be inside the square S: $0 \leq x \leq 1$ and $0 \leq y \leq 1$. (6)

(b) A small harbor has unloading facilities for ships. Only one ship can be unloaded at any one time. The unloading time required for a ship depends on the type and the amount of cargo and varies from 45 to 90 minutes.

Below is given a situation with 5 ships

	ship1	ship2	ship3	ship4	ship5
Time between successive ships (in minutes)	20	30	15	120	25
Unloading time	55	45	60	75	80

(i) Draw the time-line diagram depicting clearly the situation for each ship, the idle time for the harbor and the waiting time. (4)

(ii) List the waiting times for all the ships and find the average waiting time. (2)

(c) Using simplex method

Optimize $6x + 4y$

subject to $-x + y \leq 12$

$x + y \leq 24$

$2x + 5y \leq 80 \qquad x, y \geq 0$ (6)

(d) Explain the Linear congruence method for generating random numbers by giving a suitable example. Does this method have any drawbacks ? Illustrate. (6)

3. (a) (i) Draw a simple connected graph with degree sequence (3, 3, 3, 3, 3, 5, 5, 5). (2)

(ii) State and prove Handshaking Lemma. (4)

(b) (i) Define Hamiltonian graph. (1)

(ii) Give an example of a Hamiltonian graph. (1)

(iii) State Ore's Theorem. (2)

(iv) Let G be a simple connected graph with n vertices, where $n \geq 3$ and deg $v \geq n/2$ for each vertex v. Use Ore's theorem to show that G is Hamiltonian. (2)

(c) For which values of n, r and s are the following graphs Eulerian ? For which value are they semi-Eulerian.

(i) the complete graph K_n

(ii) the complete bipartite graph $K_{r,s}$

(iii) the n-cube Q_n (3+3)

(d) By finding an Eulerian trail in K_5, arrange a set of fifteen dominoes [from 0 – 0 to 4 – 4] in a ring. (6)

4. (a) Use the factorization $s^4 + 4a^4 = (s^2 - 2as + 2a^2)(s^2 + 2as + 2a^2)$ and apply inverse Laplace transform to show that

$$\mathcal{L}^{-1}\left\{\frac{s^3}{s^4 + 4a^4}\right\} = \cos h(at)\cos(at) \qquad (7)$$

(b) Find the Frobenius series solution of

$$xy'' + 2y' + xy = 0. \qquad (7)$$

P.T.O.

(c) Solve the problem

Maximize $25x_1 + 30x_2$

subject to $20x_1 + 30x_2 \leq 690$

$5x_1 + 4x_2 \leq 120, \quad x_1, x_2 \geq 0$ (3)

Determine the sensitivity of the optimal solution to a change in C_1 using the objective function $C_1x_1 + 30x_2$. (4)

(d) Name the five Platonic Graphs. (2.5)

What is the degree of each vertex in each of these five graphs ? (2.5)

Draw any two Platonic graphs. (2)