[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 6620 **D** **Your Roll No...............**

Unique Paper Code : 235505

Name of the Course : **B.Sc. (Hons.) MATHEMATICS**

Name of the Paper : Linear Programming and Theory of Games (Paper V.4)

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts of each questions.
3. **All** questions carry equal marks.

1. (a) Prove that to every extreme point of the feasible region, there corresponds a basic feasible solution of the linear programming problem

Minimize $z = cx$

subject to $Ax = b,\ x \geq 0.$

(b) Solve the following linear programming problem by the simplex method starting with the basic feasible solution $(x_1,x_2) = (4,0)$

$$\begin{aligned} \text{Maximize} \quad & -x_1 + 2x_2 \\ \text{subject to} \quad & 3x_1 + 4x_2 = 12 \\ & 2x_1 - x_2 \leq 12 \\ & x_1 \geq 0,\ x_2 \geq 0 \end{aligned}$$

(c) $x_1 = 1, x_2 = 1, x_3 = 1$ is a feasible solution of the system of equations

$$x_1 + x_2 + 2x_3 = 4$$

$$2x_1 - x_2 + x_3 = 2$$

Is this solution basic feasible ? If not, reduce it to a basic feasible solution.

2. (a) Using two phase method, solve the linear programming problem

Minimize $z = 3x_1 + 2x_2 + x_3$

subject to $2x_1 + 5x_2 + x_3 = 12$

$3x_1 + 4x_2 = 11$

x_1 is unrestricted, $x_2 \geq 0$, $x_3 \geq 0$.

(b) Using simplex method, solve the system of equations

$$3x_1 + 2x_2 = 4$$

$$4x_1 - x_2 = 6$$

Also, find inverse of the coefficient matrix

$$\begin{bmatrix} 3 & 2 \\ 4 & -1 \end{bmatrix}.$$

(c) Using big-M method, solve the linear programming problem

Minimize $z = -x_1 - 3x_2 + x_3$

subject to $x_1 + x_2 + 2x_3 \leq 4$

$-x_1 + x_3 \geq 4$

$x_3 \geq 3$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$

3. (a) (i) State and prove the Weak Duality Theorem.

(ii) Verify that the dual of the dual is primal.

Maximize $z = -8x_1 + 3x_2$

subject to $x_1 - 6x_2 \leq 2$

$5x_1 + 7x_2 \leq 4$

$x_1 \geq 0, \; x_2 \geq 0$

(b) Obtain the dual of the following primal problem :

Maximize $z = 7x_1 + 12x_2 + 10x_3$

subject to $3x_1 + 2x_2 - 4x_3 \leq 1$

$x_1 + 4x_2 - 3x_3 \geq 3$

$-2x_1 - 8x_3 = 2$

$x_1 \leq 0,\ x_2 \geq 0,\ x_3$ is unrestricted.

(c) Solve the following linear programming problem using duality :

Minimize $z = 2x_1 + 9x_2 + x_3$

subject to $x_1 + 4x_2 + 2x_3 \geq 5$

$3x_1 + x_2 + 2x_3 \geq 4$

$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$

4. (a) Solve the following transportation problem :

	D_1	D_2	D_3	D_4	Supply
O_1	10	2	20	11	15
O_2	12	7	9	20	25
O_3	4	14	16	18	10
Demand	5	15	15	15	

(b) Solve the following cost minimization assignment problem :

	I	II	III	IV	V
A	11	6	14	16	17
B	7	13	22	7	10
C	10	7	2	2	2
D	4	10	8	6	11
E	13	15	16	10	18

(c) (i) Find the saddle point for the game having the following pay-off matrix :

$$\begin{bmatrix} -3 & 1 & 2 \\ 1 & 2 & 1 \\ 1 & 0 & -2 \end{bmatrix}$$

(ii) Determine the range of values of p and q that will render (2,2) a saddle point for the game

$$\begin{bmatrix} 2 & 4 & 5 \\ 10 & 7 & q \\ 4 & p & 6 \end{bmatrix}$$

5. (a) Use dominance relation to reduce the following game to a 2 X 2 game, and hence find the optimum strategies and value of the game

$$\begin{bmatrix} 2 & -2 & 3 \\ -3 & 5 & -1 \\ 1 & 3 & 2 \end{bmatrix}$$

(b) Solve graphically the rectangular game whose pay-off matrix is :

$$\begin{bmatrix} 2 & 2 & 3 & -1 \\ 4 & 3 & 2 & 6 \end{bmatrix}$$

(c) Reduce the following game to a linear programming problem and then solve by simplex method.

$$\begin{bmatrix} 2 & -2 & 3 \\ -3 & 5 & -1 \end{bmatrix}$$