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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **5021**

Unique Paper Code : **237562** **D**

Name of the Paper : **STP-505/Statistical Inference**

Name of the Course : **B.Sc. (Mathematical Sciences) Statistics**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *Six* questions.

1. (*a*) Define an unbiased and consistent estimate of a parameter in a distribution. Show that (*i*) $\frac{\Sigma x_i(\Sigma x_i - 1)}{n(n-1)}$ is an unbiased estimator of θ^2, (*ii*) $\frac{\Sigma x_i}{n}\left(1 - \frac{\Sigma x_i}{n}\right)$ is a consistent estimator of $\theta(1 - \theta)$ for the sample $X_1, X_2, \ldots\ldots, X_n$ drawn on X which takes the values 1 or 0 with respective probabilities θ and $(1 - \theta)$.

   (*b*) Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample from a uniform population on $[0, \theta]$. Find a sufficient estimator for θ. 8½,4

2. (*a*) Mentioning the underlying regularity conditions, state the Cramer-Rao inequaltiy for the variance of an unbiased estimator T for $\gamma(\theta)$. Further, stating the condition for the equality sign in the Cramer-Rao inequality to hold, obtain its form.

(*b*) Let $X_1, X_2, \ldots, X_n$ be a random sample from a uniform population on $[0, \theta]$. Compute the reciprocal of $nE\left[\frac{\partial \log f(x, \theta)}{\partial \theta}\right]^2$ and compare this with the variance of $\frac{(n+1)}{n}X_{(n)}$, where $X_{(n)}$ is the largest observation in the random sample. Comment on the result. 6½,6

3. (*a*) Find the maximum likelihood estimate for the parameter λ of a Poisson distribution on the basis of a sample of size *n*. Also find its variance.

(*b*) On the basis of a random sample of size *n* drawn from Normal distribution $N(\theta, \sigma^2)$ obtain $100(1 - \alpha)\%$ confidence interval for :

(*i*) θ when σ^2 is known,

(*ii*) θ when σ^2 is unknown,

(*iii*) σ^2 when θ is known,

(*iv*) σ^2 when θ is unknown. 4½,8

4. (*a*) Let *p* be the probability that a coin will fall head in a single toss. In order to test the hypothesis $H_0 : p = \frac{1}{2}$ the coin is tossed 6 times and the hypothesis H_0 is rejected if more than 4 heads are obtained. Find the probability of error of first kind. If the alternative hypothesis is $H_1 : p = \frac{3}{4}$, find the probability of the error of second kind.

(*b*) Let $x_1, x_2, \ldots\ldots\ldots\ldots, x_n$ be a random sample of size n from Normal distribution $N(\theta, \sigma^2)$, where σ^2 is known. Obtain UMP test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta > \theta_0$. Also find the power function of the test. 4,8½

5. Explain the following terms :

(*a*) Errors of first and second kind;

(*b*) The best critical region;

(*c*) Power function of a test;

(*d*) Level of significance;

(*e*) Simple and composite hypotheses;

(*f*) Most powerful test;

(*g*) Uniformly most powerful test. 12½

6. For the normal distribution with 0 mean and variance σ^2 obtain the best critical region for testing $H_0 : \sigma = \sigma_0$ against the alternative $H_1 : \sigma = \sigma_1$. Also find the power of the best critical region when $\sigma_0 > \sigma_1$. 12½

7. (*a*) Let $x_1, x_2, \ldots, x_n$ be the observed values of the n random variables $X_1, X_2, \ldots, X_n$. Suggest a test for the hypothesis that the random variables are i.i.d.

(*b*) Let T_1 and T_2 be unbiased estimators of $\gamma(\theta)$ with efficiencies e_1 and e_2 respectively and $\rho = \rho_\theta$ be the correlation coefficient between them. Then show that :

$$\sqrt{e_1 e_2} - \sqrt{(1 - e_1)(1 - e_2)} \leq \rho \leq \sqrt{e_1 e_2} + \sqrt{(1 - e_1)(1 - e_2)}.$$ 4,8½

8. Write short notes on any *three* of the following :

(*a*) Median test;

(*b*) Difference between parametric and non-parametric tests;

(*c*) Properties of good estimators;

(*d*) Maximum likelihood method of estimation. 12½