This question paper contains 4 printed pages.

Your Roll No. ....

Sl. No. of Ques. Paper	**: 1356**	**F–7**
Unique Paper Code	**: 2351503**	
Name of Paper	**: Calculus – II (Multivariate Calculus)**	
Name of Course	**: B.Sc. (Hons.) Mathematics (Erstwhile FYUP)**	
Semester	**: V**	
Duration :	**: 3 hours**	
Maximum Marks	**: 75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

All Sections are compulsory. Attempt any five questions from each Section.

Section-I

1. (a) Let f be the function defined by :

$$f(x,y)=\begin{cases}\dfrac{xy^3}{x^2+y^6}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

Is f continuous at (0, 0)? Justify your answer.

(b) Compute the slope of the tangent line to the graph of $f(x,y)=x\ln(x+y^2)$ at the point $P_0(e,0,e)$ in the direction parallel to the xz-plane.

$\left(2\frac{1}{2}+2\frac{1}{2}\right)$

2. A closed box is found to have length 2 ft, width 4 ft and height 3 ft where the measurement of each dimension is made with a maximum possible error of ± 0.02 ft. The top of the box is made from material that costs \$ 2/ft^2 and the material for sides and bottom costs only \$ 1.50/ft^2. What is the maximum error involved in the computation of cost of the box?

(5)

3. Find $\dfrac{\partial w}{\partial s}$,

if $w=4x+y^2+z^3$, where $x=e^{rs^2}$, $y=\ln\dfrac{r+s}{t}$, and $z=rst^2$.

(5)

4. (a) Find the gradient of the function $f(x,y,z)=\dfrac{xy-1}{z+x}$.

P.T.O.

(b) Find the directional derivative of $f(x,y)=x^2+xy+y^2$ at the point $P_0(1,-1)$ in the direction towards the origin. $\left(2\frac{1}{2}+2\frac{1}{2}\right)$

5. Find the absolute extrema of the function $f(x,y)=2\sin x+5\cos y$ on the closed bounded rectangular region with vertices (0, 0), (2, 0), (2, 5) and (0, 5) **(5)**

6. Find the maximum and minimum distance from the origin to the ellipse $5x^2-6xy+5y^2=4$

(5)

Section-II

7. Sketch the region of integration and write an equivalent integral with the order of integration reversed:

$$\int_{-1}^{2}\int_{x^2-2}^{x} f(x,y)\,dy\,dx$$

(5)

8. Evaluate

$$\int_{0}^{2}\int_{0}^{\sqrt{2x-x^2}} y\sqrt{x^2+y^2}\,dy\,dx$$

by converting to polar coordinates.

(5)

9. Use a suitable change of variables to compute

$$\iint_D \exp\left(\frac{y-x}{y+x}\right)dy\,dx,$$

Where D is the triangular region with vertices (0, 0), (2, 0), (0, 2).

(5)

10. Find the volume of the tetrahedron T bounded by the plane $2x+y+3z=6$ and the coordinate planes $x=0$, $y=0$, and $z=0$ **(5)**

11. Evaluate the integral

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{x^2+y^2}^{\sqrt{2-x^2-y^2}} z\,dz\,dy\,dx$$

by transforming to cylindrical coordinates. **(5)**

12. Find the average value of the function $f(x,y,z)=x+y+z$ over the sphere $x^2+y^2+z^2=4$. **(5)**

Section-III

13. Evaluate

$$\int_C [5xy\,dx+10yz\,dy+z\,dz],$$

where C is the parabolic arc $x=y^2$ from (0, 0, 0) to (1, 1, 0) followed by the line segment given by $x=1,\ y=1,\ 0\le z\le 1$.

(5)

14. Prove that the force field $\vec{\mathbf{F}}=\left[\left(2x-x^2y\right)e^{-xy}+\tan^{-1}y\right]\hat{\mathbf{i}}+\left[\frac{x}{y^2+1}-x^3e^{-xy}\right]\hat{\mathbf{j}}$ is conservative in $\mathbf{R}^2$ and using this evaluate the line integral $\int_C \vec{\mathbf{F}}.d\vec{R}$, where C is curve with parametric equations $x=t^2\cos\pi t,\ y=e^{-t}\sin\pi t,\ 0\le t\le 1$.

(5)

15. State Green's theorem for a simply connected region in $\mathbb{R}^2$. Use it to find the work done by the force field $\vec{\mathbf{F}}(x,\ y)=(3y-4x)\hat{\mathbf{i}}+(4x-y)\hat{\mathbf{j}}$ when an object moves once counter-clockwise around the ellipse $4x^2+y^2=4$. **(5)**

16. Evaluate

$$\iint_S (x+y+z)\,dS,$$

where S is the surface defined parametrically by

$\vec{\mathbf{R}}(u,v)=(2u+v)\,\hat{\mathbf{i}}+(u-2v)\,\hat{\mathbf{j}}+(u+3v)\,\hat{\mathbf{k}}$ for $0\le u\le 1, 0\le v\le 2$. **(5)**

17. Use Stokes' theorem to evaluate

$$\oint_C [(x+2z)dx+(y-x)dy+(z-y)dz],$$

where C is the boundary of the triangular region with vertices (3, 0, 0), (0, $3/2$, 0), (0, 0, 3) traversed counter clockwise as viewed from above.

(5)

18. Use the divergence theorem to evaluate the surface integral $\iint_S \vec{\mathbf{F}}\cdot\vec{\mathbf{N}}\,dS$, where

$\vec{\mathbf{F}}(x,y,z)=\cos yz\,\hat{\mathbf{i}}+e^{xz}\,\hat{\mathbf{j}}+3z^2\,\mathbf{k}$ and S is the closed hemispherical surface

$z=\sqrt{4-x^2-y^2}$ together with the disk $x^2+y^2\le 4$ in the xy-plane and $\vec{\mathbf{N}}$ is the outward unit normal vector field.

(5)

19. State and prove the Gauss-divergence theorem.

(5)