[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1117 **G** **Your Roll No.....**

Unique Paper Code : 235501

Name of the Paper : Differential Equations and Mathematical Modelling III

Name of the Course : **B.Sc. (Hons.) Mathematics III**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **three** parts from each question.

1. (a) Solve the initial value problem using Laplace transform :

$$x'' - 6x' + 8x = 2,\ x(0) = x'(0) = 0 \qquad (6)$$

(b) (i) Find the inverse Laplace transform of (2)

$$F(s) = \frac{3}{2s-4}$$

(ii) Show that (2)

$$L\{t\ \mathrm{Cos}(kt)\} = \frac{s^2-k^2}{(s^2+k^2)^2}$$

(iii) Find the inverse Laplace transform of (2)

$$F(s) = \frac{5s-6}{s^2-3s}$$

P.T.O.

(c) Find two linearly independent Frobenius series solutions (6)

$$2xy'' - y' - y = 0$$

(d) Use power series to solve the initial value problem : (6)

$$y'' + xy' - 2y = 0, \; y(0) = 1, \; y'(0) = 0$$

2. (a) Using Monte Carlo Simulation, write an algorithm to calculate the approximation to π by considering the number of random points selected inside the quarter circle :

$$x^2 + y^2 = 1; \quad x \geq 0, \; y \geq 0,$$

when the quarter circle is taken to be inside the square S : $0 \leq x \leq 1$ and $0 \leq y \leq 1$. (6)

(b) Explain Linear Congruence method for generating random numbers. Does this method have any drawback ? Explain with the help of an example. (6)

(c) Solve the Linear Programming Problem :

Maximize : $25x_1 + 30x_2$

Subject to : $20x_1 + 30x_2 \leq 690.$

$5x_1 + 4x_2 \leq 120.$

$x_1, x_2 \geq 0.$ (6)

(d) A small harbor has unloading facilities for ships. Only one ship can be unloaded at any one time. The unloading time required for a ship depends on the type and the amount of cargo and varies from 45 to 90 minutes. Below is given a situation with 5 ships : (6)

	Ship 1	Ship 2	Ship 3	Ship 4	Ship 5
Time between successive ships (in minutes)	20	30	15	120	25
Unloading Time	35	45	60	75	80

(i) Draw the time-line diagram depicting clearly the situation for each ship, the idle time for the harbor and the waiting time.

(ii) List the waiting times for all the ships and find the average waiting time.

3. (a) (i) Draw a connected graph with 8 vertices, and a disconnected graph with 8 vertices and 2 components. (3)

(ii) Define a r - regular graph. Prove that, a r - regular graph with n vertices has nr/2 edges. (3)

(b) (i) Define Hamiltonian graph. How many Hamiltonian cycles on the dodecahedron begin with JVTSR. (4)

(ii) State Ore's Theorem. (2)

(c) Write down a Gray code of 5 – digits binary words. (6)

(d) Prove that a connected graph is semi - Eulerian iff it has exactly two vertices has odd degree. (6)

4. (a) Use the factorization : (7)

$$s^4 + 4a^4 = \left(s^2 - 2as + 2a^2\right)\left(s^2 + 2as + 2a^2\right)$$

And apply inverse Laplace transform to show that :

$$L^{-1}\left\{\frac{s^2}{s^4 + 4a^4}\right\} = \frac{1}{2a}(\cosh at \sin at + \sinh at \cos at)$$

(b) Solve the initial value problem

$$(t^2 - 2t - 3)\, y''(t) + 3(t-1)y'(t) + y = 0,\ y(1) = 4,\ y'(1) = -1$$ (7)

(c) Solve the Linear Programming Problem :

Maximize : $x_1 + 2x_2$

Subject to : $5x_1 + 2x_2 \leq 10.$

$2x_1 + 3x_2 \leq 6.$

$x_1, x_2 \geq 0.$ (7)

(d) Prove that there is no knight's tour on a 5×5 chessboard. (7)