[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1120 **G** **Your Roll No.....**

Unique Paper Code : 235505

Name of the Paper : Linear Programming and Theory of Games (Paper V.4)

Name of the Course : **B.Sc. (Hons.) MATHEMATICS**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts of each question.
3. **All** questions carry equal marks.

1. (a) Given a basic feasible solution $x_B = B^{-1}b$ with $z_0 = c_B x_B$ as the value of the objective function to the linear programming problem :

Minimize $z = c\,x$

Subject to $Ax = b$

$x \geq 0$

such that $z_j - c_j \leq 0$ for every column a_j in A. Show that z_0 is the minimum value of z subject to the constraints and that the basic feasible solution is optimal feasible solution.

(b) Prove that if there is a feasible solution to the linear programming problem :

Minimize $z = c\,x$

Subject to $Ax = b$

$x \geq 0$

then, there is a basic feasible solution to it.

P.T.O.

(c) $x_1 = 2$, $x_2 = 3$, $x_3 = 1$ is a feasible solution to the system of equations :

$$2x_1 + x_2 + 4x_3 = 11$$
$$3x_1 + x_2 + 5x_3 = 14$$

Is this a basic feasible solution ? If not, reduce it to a basic feasible solution.

2. (a) Using Two Phase method, solve the linear programming problem :

Minimize $z = x_1 - 2x_2$

Subject to $x_1 + x_2 \geq 2$

$-x_1 + x_2 \geq 1$

$x_2 \leq 3$

$x_1, x_2 \geq 0$

(b) Using simplex method, solve the system of solution :

$$x_1 + x_2 = 1$$
$$2x_1 + x_2 = 3$$

and find the inverse of the coefficient matrix $\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$.

(c) Use the big- M method to solve the following linear programming problem :

Maximize $z = 3x_1 + 2x_2 + 3x_3$

Subject to $2x_1 + x_2 + x_3 \leq 2$

$3x_1 + 4x_2 + 2x_3 \geq 8$

$x_1, x_2, x_3 \geq 0$

3. (a) (i) State and prove the relationship between the objective function values of the primal and dual linear programming problems.

(ii) Write the dual of the following linear programming problem :

Maximize $z = 8x_1 + 3x_2 - 2x_3$

Subject to $x_1 - 6x_2 + x_3 \geq 2$

$5x_1 + 7x_2 - 2x_3 = -4$

$x_1 \leq 0$, $x_2 \geq 0$, x_3 unrestricted,

and verily that dual of the dual is primal problem. (2.5,5)

(b) Verify that $(w_1, w_2) = \left(\frac{8}{5}, \frac{1}{5}\right)$

is a feasible solution of the dual of the linear programming problem :

Minimize $2x_1 + 15x_2 + 5x_3 + 6x_4$

Subject to $x_1 + 6x_2 + 3x_3 + x_4 \geq 2$

$-2x_1 + 5x_2 - x_3 + 3x_4 \leq -3$

$x_1, x_2, x_3, x_4 \geq 0$

and use this information to find the optimal solution of the primal and dual problems.

(c) Apply the principle of duality to solve the linear programming problem :

Maximize $x = 3x_1 + 2x_2$

Subject to $x_1 + x_2 \geq 1$

$x_1 + x_2 \leq 7$

$x_1 + 2x_2 \leq 10$

$x_2 \leq 3$

$x_1, x_2 \geq 0$

4. (a) Solve the following cost-minimizing transportation problem :

	D_1	D_2	D_3	Supply
O_1	10	9	8	8
O_2	10	7	10	7
O_3	11	9	7	9
O_4	12	14	10	4
Demand	10	10	8	

(b) Solve the following cost-minimizing assignment problem :

	A	B	C	D
1	18	26	17	11
II	13	28	14	26
III	38	19	18	15
IV	19	26	24	10

(c) Define saddle point of a two person zero sum game. Use the minimax criteria to find the best strategy for each player for the game having the following pay off matrix :

$$\begin{array}{cc} & \text{Player II} \\ \text{Player I} & \begin{bmatrix} 1 & -1 \\ -2 & 0 \\ 3 & 1 \end{bmatrix} \end{array}$$

Is it a stable game ? (3.5,4)

5. (a) Solve graphically the game whose payoff matrix is

$$\begin{bmatrix} 2 & 4 & 11 \\ 7 & 4 & 2 \end{bmatrix}$$

(b) Use the relation of dominance to solve the game whose payoff matrix is given by

$$\begin{bmatrix} 1 & 7 & 3 & 4 \\ 5 & 6 & 4 & 5 \\ 7 & 2 & 0 & 3 \end{bmatrix}$$

(c) Reduce the following game to a Linear Programming Problem and then solve by simplex method.

$$\begin{bmatrix} 1 & -3 & 2 \\ -4 & 4 & -2 \end{bmatrix}$$

(2800)