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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **32**

Unique Paper Code : **235566** **G**

Name of the Paper : **MAPT-505, Real Analysis**

Name of the Course : **B.Sc. Mathematical Sciences/B.Sc. Physical Sciences**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any *two* parts from each question.

1. (*a*) State order completeness property. Show that the set of irrational numbers is not order complete.

   (*b*) State and prove Archimedean property of real numbers.

   (*c*) Show that the set of positive rational numbers is countable. 6,6

2. (*a*) Prove that the set $S = \{1/n | n \in N\}$ has no limit point other than zero in the set of real numbers.

   (*b*) Show that every subset of a countable set is countable. Is every superset of a countable set countable ? Justify.

   (*c*) If $\lim a_n = a$ and $a_n \geq 0$ for all n, then prove that $a \geq 0$. Deduce that if $\{a_n\}$ and $\{b_n\}$ are two sequences such that $a_n \geq b_n$ for all n, then $\lim a_n \geq \lim b_n$. 6,6

P.T.O.

3. (*a*) State and prove Bolzano-Weierstrass theorem for sequences.

(*b*) Define a Cauchy sequence. Show that the sequence $\{a_n\}$ is not convergent, where :

$$a_n = 1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \ldots .$$

(*c*) Let $\{x_n\}$ be a sequence defined by :

$$x_1 = 1,\ x_{n+1} = \frac{3+2a_n}{2+a_n},\ n \geq 1.$$

Show that the sequence $\{x_n\}$ is convergent. Also find its limit. 6½,6½

4. (*a*) Define the convergence of an infinite series. Show that the series :

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots..$$

is convergent.

(*b*) Test the convergence of the following series :

(*i*) $\sum \frac{\sqrt{n+1} - \sqrt{n-1}}{n}$

(*ii*) $1 + \frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \ldots\ldots$ for all $x > 0$.

(*c*) Let $\Sigma\, u_n$ be a positive term series such that

$$\lim_{n \to \infty} (u_n)^{\frac{1}{n}} = \text{L}.$$

Show that $\Sigma\, u_n$ converges for $\text{L} < 1$. What happens for $\text{L} = 1$? Justify. 6,6

5. (*a*) State Leibnitz test for convergence of an alternating series. Test for convergence and absolute convergence, the series :

$$1 - \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} - \frac{1}{4\sqrt{4}} + \ldots .$$

Is the series conditionally convergent ?

(*b*) Define radius of convergence and interval of convergence of a power series. Find the radius of convergence and exact interval of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{(x-1)^n}{2^n}.$$

(*c*) Define sine function S(x) and cosine function C(x) in terms of power series. Specify the domains of convergence of the respective power series. 6½, 6½

6. (*a*) If $\{f_n\}$ is a sequence of continuous functions converging uniformly to a function f on $[a, b]$, then show that f is continuous on $[a, b]$.

(*b*) Test the sequence $\{f_n\}$ where

$$f_n(x) = \frac{nx}{1+n^2x^2}$$

for uniform convergence on [0, 1]. Verify :

$$\lim_{n\to\infty} \int_0^1 f_n(x)\,dx = \int_0^1 \left(\lim_{n\to\infty} f_n(x)\right) dx.$$

(*c*) (*i*) Show that the series :

$$\sum \frac{1}{n^4 + n^2x^2}$$

converges uniformly for all real values of x.

(*ii*) Show that the series :

$$x^4 + \frac{x^4}{1+x^4} + \frac{x^4}{\left(1+x^4\right)^2} + \frac{x^4}{\left(1+x^4\right)^3} + \dots$$

is not uniformly convergent on [0, 1]. 6½,6½