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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **30**

Unique Paper Code : **237562** **G**

Name of the Paper : **Statistical Inference**

Name of the Course : **B.Sc. (Mathematical Sciences) Statistics**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *six* questions.

All questions carry equal marks.

1. (*a*) Explain the following terms :

(*i*) Type I and Type II errors

(*ii*) Simple and composite hypothesis.

(*b*) Define a sufficient statistic for the unknown parameter $\gamma(\theta)$. Let $x_1, x_2, ..., x_n$ be a random sample of size n from normal distribution $N(\mu, \sigma^2)$. Find sufficient estimators for μ and σ^2.

2. (*a*) Let T_1 and T_2 be the two estimators of $\gamma(\theta)$ with variances σ_1^2, σ_2^2 and correlation coefficient ρ. What is the best unbiased linear combination of T_1 and T_2 ? Find the variance of such a combination.

P.T.O.

(*b*) Develop a general method for constructing confidence interval. Construct a $100(1-\alpha)\%$ confidence interval for the difference of means of two normal populations with an unknown but common variance, given two independent samples of sizes n_1 and n_2 from the two populations $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$.

3. (*a*) In random sampling from a normal population $N(\mu, \sigma^2)$, find the maximum Likelihood Estimators for :

   (*i*) μ when σ^2 is known

   (*ii*) σ^2 when μ is known

   (*iii*) μ and σ^2.

   (*b*) Let p be the probability that a coin shows head in a single toss in order to test

$$H_0 : p = \frac{1}{2} \text{ against } H_1 : p = \frac{3}{4}.$$

   The coin is tossed 6 times and H_0 is rejected if more than 4 heads are obtained. Find the probability of Type I error and power of the test.

4. Let $x_1, x_2, \ldots, x_n$ be a random sample of size n from normal distribution $N(\theta, \sigma^2)$ where σ^2 is known. Obtain UMP test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta > \theta_0$. Also find the power function of the test.

5. (*a*) Describe in brief the method of moments for estimating the parameters.

(*b*) For the distribution

$$dF = \begin{cases} \beta \exp\left(-\beta(x-\gamma)\right)dx, & x \geq \gamma \\ 0 & , x < \gamma \end{cases}$$

show that for a null hypothesis

$$H_0 : \beta = \beta_0, \gamma = \gamma_0$$

and an alternative

$$H_1 : \beta = \beta_1, \gamma = \gamma_1,$$

the best critical region is given by :

$$\bar{x} \leq \frac{1}{\beta_1 - \beta_0}\left\{\gamma_1\beta_1 - \gamma_0\beta_0 - \frac{1}{n}\log k + \log\frac{\beta_1}{\beta_0}\right\}$$

provided that admissible hypothesis is restricted by the condition $\beta_1 \geq \beta_0, \gamma_0 \geq \gamma_1$.

6. (*a*) What are the advantages and disadvantages of the parametric and the non-parametric approaches to the theory of statistical inference ?

(*b*) A random sample $x_1, x_2, ..., x_n$ is drawn from a normal population with mean μ and variance σ^2, (σ^2 is known). Obtain an MVB estimator for μ.

7. (*a*) What do you understand by point estimation ? When would you say that estimate of a parameter is good ? In particular, discuss any *two* properties of an estimator in detail, giving *one* example for each.

(*b*) Let $x_1, x_2, ..., x_n$ be the observed values of the n random variables $X_1, X_2, ..., X_n$. Develop run test for testing the hypothesis that the random variables are i.i.d.

8. Write short notes on any *two* of the following :

(*a*) Properties of good estimators

(*b*) Maximum likelihood method of estimation

(*c*) Median test.